

FIRE JUNGLE

CAPS-aligned mathematics for South African learners



GRADE 11

Mathematics

TEXTBOOK

Definitions • Worked Examples • Exercises • Full Answers



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by Joseph Matthew Mitchell

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Fire Jungle Grade 11 Mathematics

A CAPS-aligned study textbook for the FET phase

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How to use this book

Definitions. Each topic opens with the key terms you must know and be able to state in an exam.

Key formulas. The formulas and rules for the topic, gathered in one box for quick revision.

Worked examples. Fully solved problems. The right-hand notes explain the reason for every step.

Exercises. Practice questions. Try them before checking the back of the book.

Answers. Complete answers with working are collected in the Answers section at the end.

Digital edition. Scan the QR code on any page to open the interactive version of this textbook, with animated lessons, narration and unlimited generated practice.

Tip: work through the worked examples with a pen in hand, then attempt the exercises with the book closed. Use the digital edition for extra randomised practice on any topic you find hard.



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CHAPTER 2

Equations and inequalities

- 2.1 Quadratics by factorising
- 2.2 Completing the square
- 2.3 The quadratic formula
- 2.4 Substitution / k-method
- 2.5 Equation from roots
- 2.6 Nature of roots (discriminant)
- 2.7 Quadratic inequalities
- 2.8 Simultaneous: 1 linear + 1 quadratic
- 2.9 Quadratic word problems

2.1 Revision: solving quadratics by factorising

DEFINITIONS

Quadratic equation	An equation of the form $ax^2 + bx + c = 0$.
Factorising	Writing the quadratic as a product of two brackets that equal zero.

KEY FORMULAS

- **Standard form:** $ax^2 + bx + c = 0$
- **Zero-product rule:** if $pq = 0$ then $p = 0$ or $q = 0$
- **Difference of squares:** $a^2 - b^2 = (a - b)(a + b)$
- **Common factor first**, then split the middle term

WORKED EXAMPLES

Example 1: Solve $x^2 - 7x + 12 = 0$.

- Find two numbers that multiply to 12 and add to -7 : -3 and -4 .

Build the factors from those numbers.

- $(x - 3)(x - 4) = 0$

Factorised form.

- $x = 3$ or $x = 4$

Zero-product rule.



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Applied example — A garden width

A rectangular garden is 3 m longer than it is wide and has area 40 m². Find its width.

► $w(w + 3) = 40 \rightarrow w^2 + 3w - 40 = 0$

Set up the area equation.

► $(w + 8)(w - 5) = 0 \rightarrow w = 5 \text{ m}$

Factorise; reject the negative width.

EXERCISES (EASY → DIFFICULT)

1. Solve $x^2 - 5x + 6 = 0$.
2. Solve $x^2 - 9 = 0$.
3. Solve $x^2 + 2x - 8 = 0$.
4. Solve $x^2 - 7x + 12 = 0$.
5. Solve $2x^2 - 5x - 3 = 0$.
6. The product of two consecutive integers is 56. Find them.
7. A number subtracted from its square gives 12. Find the number.
8. A rectangular garden is 3 m longer than it is wide and has area 40 m². Find its width.
9. A photo has area 36 cm² and is 5 cm longer than it is wide. Find its width.
10. A triangle's base is 2 cm longer than its height and its area is 24 cm². Find the height.

2.2 Completing the square

DEFINITIONS

Completing the square Rewriting $ax^2 + bx + c$ as $a(x + p)^2 + q$ to reveal the vertex.

Perfect square An expression like $(x + p)^2$ that expands to $x^2 + 2px + p^2$.

KEY FORMULAS

- **Step 1:** Move the constant: $x^2 + bx = -c$
- **Step 2:** Add $(b/2)^2$ to both sides
- **Result:** $(x + b/2)^2 = (b/2)^2 - c$
- **Step 3:** Square-root both sides (don't forget $\pm$)

WORKED EXAMPLES

Example 1: Solve $x^2 + 4x - 5 = 0$ by completing the square.

► Move -5 : $x^2 + 4x = 5$.

Get the constant onto the right.

► Add $(4/2)^2 = 4$ to both sides: $x^2 + 4x + 4 = 9$.

Now the left is a perfect square.

► $(x + 2)^2 = 9 \rightarrow x + 2 = \pm 3 \rightarrow x = 1 \text{ or } x = -5$.

Take $\pm$ square root and isolate x .



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**Applied example — Maximum profit**

A stall's profit is $P = -x^2 + 10x - 9$ (R1000s). Find the maximum profit by completing the square.

► $P = -(x^2 - 10x) - 9 = -(x - 5)^2 + 16$

Complete the square.

► Maximum profit = R16 000 at $x = 5$

The vertex gives the maximum.

EXERCISES (EASY → DIFFICULT)

1. Complete the square: $x^2 + 6x + \underline{\hspace{1cm}}$.
2. Complete the square: $x^2 - 10x + \underline{\hspace{1cm}}$.
3. Solve $x^2 + 4x - 5 = 0$ by completing the square.
4. Solve $x^2 - 2x - 3 = 0$ by completing the square.
5. Solve $x^2 - 4x + 1 = 0$ by completing the square.
6. Write $y = x^2 - 6x + 5$ in the form $(x - p)^2 + q$ and state the turning point.
7. Write $y = x^2 + 2x + 5$ in vertex form and give the minimum value.
8. A ball's height is $h = 4t - t^2$. By completing the square, find its maximum height.
9. A stall's profit is $P = -x^2 + 10x - 9$. Find the maximum profit by completing the square.
10. Solve $2x^2 - 8x + 3 = 0$ by completing the square.

2.3 Quadratic formula

The quadratic formula

DEFINITIONS**Quadratic formula**

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$ — solves any quadratic.

Discriminant

$b^2 - 4ac$, the part under the root sign.

KEY FORMULAS

- **Formula:** $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$
- **Discriminant:** $\Delta = b^2 - 4ac$
- $\Delta > 0 \rightarrow$ two real roots
- $\Delta = 0 \rightarrow$ one real (repeated) root
- $\Delta < 0 \rightarrow$ no real roots

WORKED EXAMPLES**Example 1: Solve $2x^2 + 5x - 3 = 0$ using the formula.**

► $a = 2, b = 5, c = -3$. Discriminant = $25 + 24 = 49$.

Compute Δ first — saves work later.

► $x = (-5 \pm 7) / 4$

Plug into the formula.

► $x = 1/2$ or $x = -3$

Two real solutions because $\Delta > 0$.

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**Applied example — Width of a border**

A picture 8 cm by 10 cm has a uniform border of width x . The total area is 143 cm^2 . Find x .

► $(8 + 2x)(10 + 2x) = 143$

Total area = picture + border.

► $4x^2 + 36x - 63 = 0$

Expand and simplify.

► $x = (-36 + \sqrt{2304})/8 = 1.5 \text{ cm}$

Quadratic formula; take the positive root.

EXERCISES (EASY → DIFFICULT)

1. Solve $x^2 - 3x - 4 = 0$.
2. Solve $x^2 + x - 6 = 0$.
3. Solve $2x^2 - 7x + 3 = 0$.
4. Solve $x^2 - 4x + 1 = 0$ using the quadratic formula (2 dp).
5. Solve $3x^2 - 2x - 2 = 0$ (2 dp).
6. A stone's height is $h = 25t - 5t^2 \text{ m}$. When is it at 20 m?
7. A rectangular plot has area 50 m^2 and is 3 m longer than it is wide. Find its width (2 dp).
8. Solve $x^2 - 6x + 7 = 0$ (leave in surd form).
9. Solve $2x^2 + 3x - 4 = 0$ (2 dp).
10. A picture 8 cm by 10 cm has a uniform border of width x . The total area is 143 cm^2 . Find x .

2.4 Substitution (k-method)

Substitution / k-method

DEFINITIONS

Substitution (k-method) Replacing a repeated expression with a single letter k to simplify.

Variable A symbol standing for an unknown number.

KEY FORMULAS

- When you see **repeated patterns**, substitute a single letter for the pattern.
- e.g. $(x^2 - 3)^2 + (x^2 - 3) - 6 = 0 \rightarrow \text{let } k = x^2 - 3$

WORKED EXAMPLES

Example 1: Solve $(x - 1)^2 - 5(x - 1) + 6 = 0$.

► Let $k = x - 1$. Then $k^2 - 5k + 6 = 0$.

Repeated chunk: $(x - 1)$.

► $(k - 2)(k - 3) = 0 \rightarrow k = 2 \text{ or } k = 3$.

Standard factorisation.

► $x - 1 = 2 \rightarrow x = 3$. $x - 1 = 3 \rightarrow x = 4$.

Substitute back.

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**Applied example — The k-method**

Solve $x^4 - 5x^2 + 4 = 0$ by substitution.

► Let $k = x^2$: $k^2 - 5k + 4 = 0$

Substitute to make it quadratic.

► $(k - 1)(k - 4) = 0 \rightarrow k = 1$ or 4

Factorise.

► $x^2 = 1$ or $4 \rightarrow x = \pm 1, \pm 2$

Replace k and solve.

EXERCISES (EASY → DIFFICULT)

1. Solve $x^4 - 5x^2 + 4 = 0$ (let $k = x^2$).
2. Solve $x^4 - 13x^2 + 36 = 0$.
3. Solve $x^4 - 10x^2 + 9 = 0$.
4. Let $k = x^2$ in $2x^4 - 9x^2 + 4 = 0$ and solve.
5. Solve $(x^2 - x)^2 - 8(x^2 - x) + 12 = 0$ (let $k = x^2 - x$).
6. Solve $x - 7\sqrt{x} + 12 = 0$ (let $k = \sqrt{x}$).
7. Solve $x + \sqrt{x} - 6 = 0$ (let $k = \sqrt{x}$).
8. Solve $(2x - 1)^2 - 5(2x - 1) + 6 = 0$ (let $k = 2x - 1$).
9. Solve $2^{(2x)} - 5 \cdot 2^x + 4 = 0$ (let $k = 2^x$).
10. Solve $(x + 1/x)^2 - 4(x + 1/x) + 3 = 0$ (let $k = x + 1/x$).

2.5 Finding the equation given roots

Equation from roots

DEFINITIONS

Sum of roots For $x^2 - (\text{sum})x + (\text{product}) = 0$, the roots add to the middle coefficient.

Product of roots The roots multiply to give the constant term.

KEY FORMULAS

- If the roots are **p** and **q**, the quadratic is
- **$(x - p)(x - q) = 0$**
- Expanded: $x^2 - (p + q)x + pq = 0$
- **Sum of roots** = $-b/a$, **product** = c/a

WORKED EXAMPLES**Example 1: Find the quadratic whose roots are -3 and 4.**

► $(x - (-3))(x - 4) = (x + 3)(x - 4)$

Substitute each root into the factor form.

► Expand: $x^2 - x - 12 = 0$.

FOIL it out.

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**Applied example — Equation from roots**

Find a quadratic equation whose roots are $1 + \sqrt{2}$ and $1 - \sqrt{2}$.

► Sum = 2, product = $(1)^2 - (\sqrt{2})^2 = -1$

Sum and product of roots.

► $x^2 - (\text{sum})x + \text{product} = 0 \rightarrow x^2 - 2x - 1 = 0$

Build the equation.

EXERCISES (EASY → DIFFICULT)

- Find a quadratic equation with roots 2 and 3.
- Find a quadratic with roots -1 and 4.
- Find a quadratic with roots -2 and -5 .
- Find a quadratic with roots 0 and 6.
- Find a quadratic with a double root at 3.
- A quadratic has roots that sum to 7 and multiply to 10. Write the equation.
- Find a quadratic with roots $1 + \sqrt{2}$ and $1 - \sqrt{2}$.
- Find a quadratic with roots $\frac{1}{2}$ and -3 .
- One root of $x^2 + bx + 12 = 0$ is 3. Find the other root and b .
- The roots of a quadratic are reciprocals of the roots of $x^2 - 5x + 6 = 0$. Find the new equation.

2.6 Nature of roots (discriminant)**DEFINITIONS**

Discriminant Δ $\Delta = b^2 - 4ac$ decides the nature of the roots.

Nature of roots $\Delta > 0$: two real; $\Delta = 0$: one repeated; $\Delta < 0$: none real.

KEY FORMULAS

- $\Delta = b^2 - 4ac$
- $\Delta > 0 \rightarrow$ two unequal real roots
- $\Delta = 0 \rightarrow$ one repeated (equal) real root
- $\Delta < 0 \rightarrow$ no real roots (complex roots)
- Δ is a perfect square $\Rightarrow$ roots are **rational**

WORKED EXAMPLES

Example 1: What is the nature of the roots of $3x^2 - 4x + 2 = 0$?

► $\Delta = (-4)^2 - 4(3)(2) = 16 - 24 = -8$.

Substitute $a=3$, $b=-4$, $c=2$.

► $\Delta < 0$, so there are **no real roots**.

The parabola does not cross the x-axis.

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Applied example — Equal roots

For which value of k does $x^2 + kx + 9 = 0$ have equal roots?

► $\Delta = k^2 - 4(9) = 0$

Equal roots $\Rightarrow$ discriminant $= 0$.

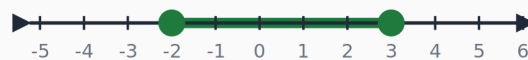
► $k^2 = 36 \rightarrow k = \pm 6$

Solve.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. State the discriminant of $ax^2 + bx + c = 0$.
2. Find Δ for $x^2 - 4x + 4 = 0$ and describe the roots.
3. Find Δ for $x^2 + x + 1 = 0$ and describe the roots.
4. Find Δ for $x^2 - 5x + 6 = 0$ and describe the roots.
5. Find Δ for $2x^2 - 3x - 1 = 0$ and describe the roots.
6. For which value of k does $x^2 + kx + 9 = 0$ have equal roots?
7. For which values of k does $x^2 - 4x + k = 0$ have real roots?
8. Show that $x^2 - 2x + 3 = 0$ has no real roots.
9. For which k does $2x^2 + 4x + k = 0$ have non-real roots?
10. Show that $x^2 - (m + 1)x + m = 0$ always has real roots for all real m .

2.7 Quadratic inequalities



closed dots: -2 and 3 ARE included

$$x^2 - x - 6 \leq 0 \Rightarrow -2 \leq x \leq 3$$

DEFINITIONS

Quadratic inequality

An inequality like $ax^2 + bx + c > 0$ — solved using a number line.

Critical values

The roots, which split the number line into regions.

KEY FORMULAS

- Bring the inequality to the form $ax^2 + bx + c \oplus 0$ ($\oplus$ is one of $<$, $\leq$, $>$, $\geq$)
- Find the roots — they split the number line into regions.
- Test ONE value from each region to see if the inequality holds.
- For **parabola opens up** ($a > 0$): expression ≤ 0 BETWEEN the roots.



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WORKED EXAMPLES

Example 1: Solve $x^2 + 2x - 8 < 0$.

► Factorise: $(x + 4)(x - 2) < 0$.

Roots are $x = -4$ and $x = 2$.

► Parabola opens up — it's below zero BETWEEN the roots.

Test $x = 0$: $(0+4)(0-2) = -8 < 0$.

► $-4 < x < 2$

Strict inequality, so don't include the roots.

Applied example — When is the firm profitable?

A profit (R1000s) is $P = (x - 2)(x - 6)$. For which output x is $P > 0$?

► $(x - 2)(x - 6) > 0$

Profit positive.

► $x < 2$ or $x > 6$

Product positive outside the roots.

EXERCISES (EASY → DIFFICULT)

1. Solve $x - 3 > 0$.
2. Solve $(x - 1)(x - 4) > 0$.
3. Solve $(x + 2)(x - 3) < 0$.
4. Solve $x^2 - 9 \leq 0$.
5. Solve $x^2 - 5x + 6 > 0$.
6. A profit (R1000s) is $P = (x - 2)(x - 6)$. For which production levels x is the firm profitable ($P > 0$)?
7. A projectile is above ground while $-x^2 + 4x > 0$. For which x ?
8. Solve $x^2 \geq 4x$.
9. Solve $x^2 - 2x - 8 \leq 0$.
10. Solve $(x - 1)^2(x + 3) > 0$ (state where it holds).

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2.8 Simultaneous (1 linear + 1 quadratic)

Simultaneous: 1 linear + 1 quadratic

DEFINITIONS

Simultaneous equations A set of equations solved together for common solutions.

Substitution method Solve the linear equation for one variable, then substitute.

KEY FORMULAS

- From the LINEAR equation, make **y** the subject (or x).
- Substitute into the QUADRATIC. You'll get a quadratic in one variable.
- Solve, then back-substitute to find the other variable.

WORKED EXAMPLES

Example 1: Solve simultaneously: $y = x + 2$ and $y = x^2 - 4$.

► Set the y's equal: $x + 2 = x^2 - 4$.

Both equal y.

► $x^2 - x - 6 = 0 \rightarrow (x - 3)(x + 2) = 0 \rightarrow x = 3$ or -2 .

Standard factorising.

► Back-sub: if $x = 3$, $y = 5$; if $x = -2$, $y = 0$.

Two intersection points: (3, 5) and (-2, 0).

Applied example — Rectangle from perimeter & area

A rectangle has perimeter 26 m and area 40 m². Find its dimensions.

► $x + y = 13$ and $xy = 40$

Half the perimeter and the area.

► $x^2 - 13x + 40 = 0 \rightarrow (x - 5)(x - 8) = 0$

Substitute $y = 13 - x$.

► 5 m $\times$ 8 m

The two sides.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. Solve $y = x + 1$ and $y = x^2$ simultaneously.
2. Solve $y = 2x$ and $y = x^2 - 3$.
3. Solve $y = x - 1$ and $y = x^2 - 3x + 2$.
4. Solve $x + y = 5$ and $xy = 6$.
5. Solve $y = x + 2$ and $x^2 + y^2 = 10$.
6. The sum of two numbers is 10 and their product is 21. Find them.
7. A line $y = x + 1$ meets the parabola $y = x^2 - x - 2$. Find the points.
8. Two numbers differ by 4 and have product 45. Find them.
9. Solve $2x - y = 1$ and $y = x^2 - 2$.
10. A rectangle has perimeter 26 m and area 40 m². Find its dimensions.



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2.9 Word problems (quadratic)

Quadratic word problems

DEFINITIONS

Word problem A real-life situation translated into a quadratic equation.

Variable A letter chosen to represent the unknown quantity.

KEY FORMULAS

- 1. Define your variable clearly.
- 2. Translate the words into an equation.
- 3. Solve the quadratic.
- 4. **Reject solutions** that don't make physical sense (e.g. negative lengths or times).

WORKED EXAMPLES

Example 1: The sum of a number and its square is 30. Find the number.

► Let x = the number. Then $x + x^2 = 30$.

Translate the words.

► $x^2 + x - 30 = 0 \rightarrow (x + 6)(x - 5) = 0 \rightarrow x = -6$ or 5 .

Factorise.

► Both are valid here (no physical restriction).

For "the number", either solution works.

Applied example — Sharing money

Learners share R120 equally. With 2 more learners each gets R2 less. How many learners are there?

► $120/n - 120/(n + 2) = 2$

Difference in each share.

► $n^2 + 2n - 120 = 0 \rightarrow (n + 12)(n - 10) = 0$

Simplify and factorise.

► $n = 10$ learners

Reject the negative root.

EXERCISES (EASY → DIFFICULT)

1. The product of two consecutive even numbers is 80. Find them.
2. A number plus its square is 30. Find the positive number.
3. A rectangular field is 5 m longer than it is wide, with area 84 m^2 . Find its dimensions.
4. The sum of the squares of two consecutive integers is 25. Find them.
5. A ball is thrown up; its height is $h = 30t - 5t^2$ m. When does it hit the ground again?
6. Lerato is 3 years older than Sipho. The product of their ages is 70. How old is Sipho?
7. A rectangular pool is surrounded by a 1 m path. The pool is 6 m by 4 m. The total area (pool + path) is...
8. A garden path of uniform width x surrounds a $10 \text{ m} \times 6 \text{ m}$ lawn, making total area 96 m^2 . Find x .
9. A stone dropped into a well takes t seconds; distance $d = 5t^2$. If it falls 45 m, find t .
10. A number of learners share R120 equally. If there were 2 more learners, each would get R2 less. How many learners are there?



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End of Chapter 2 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Solve $x^2 - 5x + 6 = 0$.
2. Complete the square: $x^2 - 10x + \underline{\hspace{1cm}}$.
3. Solve $2x^2 - 7x + 3 = 0$.
4. Let $k = x^2$ in $2x^4 - 9x^2 + 4 = 0$ and solve.
5. Find a quadratic with a double root at 3.
6. For which value of k does $x^2 + kx + 9 = 0$ have equal roots?
7. A projectile is above ground while $-x^2 + 4x > 0$. For which x ?
8. Two numbers differ by 4 and have product 45. Find them.
9. A stone dropped into a well takes t seconds; distance $d = 5t^2$. If it falls 45 m, find t .
10. A triangle's base is 2 cm longer than its height and its area is 24 cm^2 . Find the height.

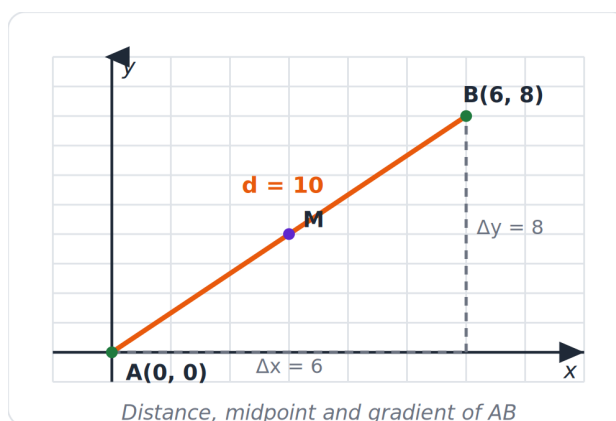


CHAPTER 4

Analytical geometry

- 4.1 Revision: distance, midpoint, gradient
- 4.2 Equation of a line
- 4.3 Inclination of a line
- 4.4 Parallel lines
- 4.5 Perpendicular lines

4.1 Revision: distance, midpoint, gradient



DEFINITIONS

Distance formula	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
Midpoint	$M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$.
Gradient	$m = (y_2 - y_1)/(x_2 - x_1)$, the steepness of a line.

KEY FORMULAS

- **Distance:** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- **Midpoint:** $M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$
- **Gradient:** $m = (y_2 - y_1) / (x_2 - x_1)$



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WORKED EXAMPLES

Example 1: Find the distance, midpoint and gradient of A(1, 2) and B(5, 8).

► $d = \sqrt{(4^2 + 6^2)} = \sqrt{52} \approx 7.21$.

Pythagoras.

► $M = (3, 5)$.

Average each coordinate.

► $m = 6/4 = 1.5$.

Rise over run.

Applied example — Distance on a town map

On a town grid (km), a school is at (2, 3) and a clinic at (5, 7). How far apart are they?

► $d = \sqrt{(5-2)^2 + (7-3)^2}$

Distance formula.

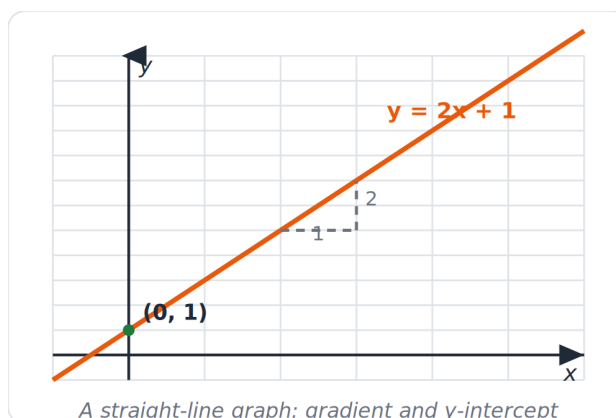
► $= \sqrt{(9 + 16)} = 5 \text{ km}$

Evaluate.

EXERCISES (EASY → DIFFICULT)

- Find the distance between (0, 0) and (3, 4).
- Find the midpoint of (2, 4) and (6, 8).
- Find the gradient of the line through (1, 2) and (3, 8).
- Find the distance between (1, 2) and (4, 6).
- Find the midpoint of (−2, 3) and (4, −1).
- On a town map (km), a school is at (2, 3) and a clinic at (5, 7). How far apart are they?
- A straight road runs from (1, 1) to (5, 9). Find its gradient (steepness).
- Find the midpoint of a fence from post A(−3, 2) to post B(7, −4) (where to put the gate).
- A drone flies from (−2, 1) to (1, 5) (km). How far does it fly?
- Show that (1, 2), (4, 3) and (7, 4) are collinear (same line).

4.2 Equation of a line



DEFINITIONS

Equation of a line $y = mx + c$, where m is the gradient and c the y-intercept.

y-intercept Where the line crosses the y-axis ($x = 0$).



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KEY FORMULAS

- **Slope-intercept:** $y = mx + c$
- **Point-slope:** $y - y_1 = m(x - x_1)$
- Two points → find m first, then point-slope.

WORKED EXAMPLES

Example 1: Find the equation of the line through (2, 1) with gradient 3.

- Point-slope: $y - 1 = 3(x - 2)$.

Substitute directly.

- Expand: $y = 3x - 5$.

Slope-intercept form.

Applied example — A taxi-fare line

A taxi charges R12 plus R8 per km. Write the fare as a line and find the fare for 9 km.

- $y = 8x + 12$

Fixed charge is the y-intercept; rate is the gradient.

- $y = 8(9) + 12 = \text{R}84$

Substitute $x = 9$.

EXERCISES (EASY → DIFFICULT)

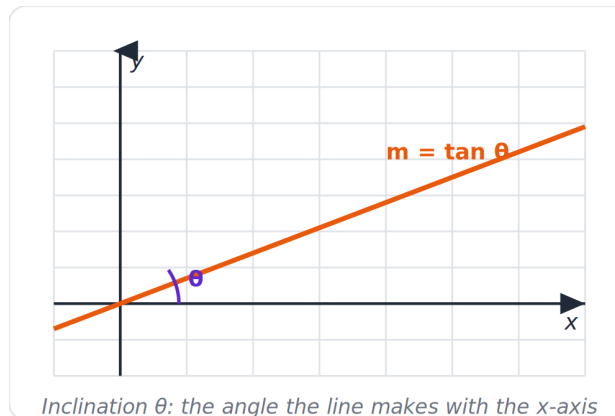
1. Find the equation of the line with gradient 2 and y-intercept -3 .
2. Find the equation of the line with gradient -1 through $(0, 4)$.
3. Find the equation of the line through $(1, 2)$ with gradient 3.
4. Find the equation of the line through $(2, 5)$ and $(4, 9)$.
5. Find the equation of the line through $(0, 0)$ and $(3, 6)$.
6. A taxi fare is R12 plus R8 per km. Write the fare y as a function of distance x .
7. A plumber charges a R350 call-out plus R250 per hour. Write the cost as a line.
8. A line passes through $(1, 4)$ and $(3, 10)$. Find its equation.
9. Find the x- and y-intercepts of $2x + 3y = 12$.
10. A candle burns down: after 0 h it is 20 cm, after 4 h it is 12 cm. Write its height as a line.



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4.3 Inclination of a line



DEFINITIONS

Inclination The angle θ a line makes with the positive x-axis.

Gradient & angle $m = \tan \theta$.

KEY FORMULAS

- **Inclination θ :** angle between the line and the positive x-axis ($0^\circ \leq \theta < 180^\circ$)
- **$\tan \theta = m$** (gradient = tan of inclination)
- If $m < 0$, θ is obtuse ($90^\circ < \theta < 180^\circ$)

WORKED EXAMPLES

Example 1: Find the inclination of the line $y = 2x - 5$.

► $m = 2$, so $\tan \theta = 2$.

Gradient is the coefficient of x .

► $\theta = \tan^{-1}(2) \approx 63.43^\circ$.

Positive gradient $\rightarrow$ acute inclination.

Applied example — Angle of a ramp

A wheelchair ramp rises 1 m over a horizontal run of 4 m. Find its angle of inclination.

► $\tan \theta = \text{rise/run} = 1/4$

Gradient equals tan of the inclination.

► $\theta = \tan^{-1}(0.25) \approx 14.04^\circ$

Evaluate.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. Find the gradient of a line with inclination 45° .
2. Find the inclination of a line with gradient 1.
3. Find the inclination of a line with gradient $\sqrt{3}$.
4. Find the gradient of a line with inclination 30° .
5. Find the inclination of a horizontal line.
6. A wheelchair ramp rises 1 m over 4 m horizontally. Find its angle of inclination.
7. A road has gradient 2. Find its angle of inclination.
8. Find the inclination of the line through (1, 1) and (4, 4).

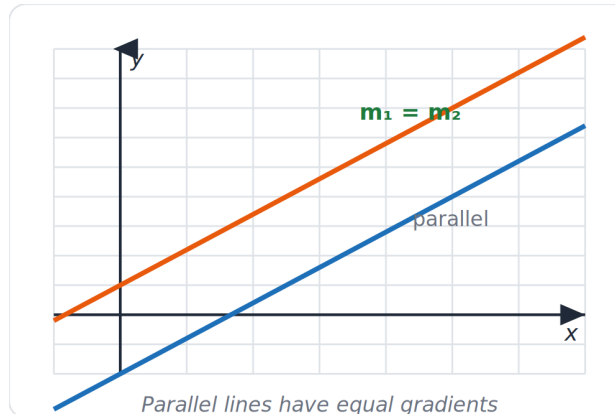


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9. A line has inclination 135° . Find its gradient.
10. A ladder leans so its foot is 2 m from a wall and its top is 5 m up. Find its angle with the ground.

4.4 Parallel lines



DEFINITIONS

Simultaneous equations	Two or more equations that must hold AT THE SAME TIME for the same set of unknowns.
Substitution	Make one variable the subject in one equation, then substitute into the other.
Elimination	Add or subtract the equations to remove one variable.

KEY FORMULAS

- Two lines are **parallel** $\Leftrightarrow$ they have the same gradient.
- If line₁: $y = m_1x + c_1$ and line₂: $y = m_2x + c_2$, then $m_1 = m_2$.

WORKED EXAMPLES

Example 1: Find the line through (3, 1) parallel to $y = -2x + 4$.

- Gradient of the given line is -2 , so the new line also has $m = -2$.

Same slope.

- Point-slope: $y - 1 = -2(x - 3) \rightarrow y = -2x + 7$.

Use the given point.

Applied example — Keeping tracks parallel

A railway track follows $y = \frac{1}{2}x + 3$. Find the parallel track through (4, 1).

- Parallel $\Rightarrow$ same gradient $\frac{1}{2}$

Equal gradients.

- $y - 1 = \frac{1}{2}(x - 4) \rightarrow y = \frac{1}{2}x - 1$

Point-gradient form.



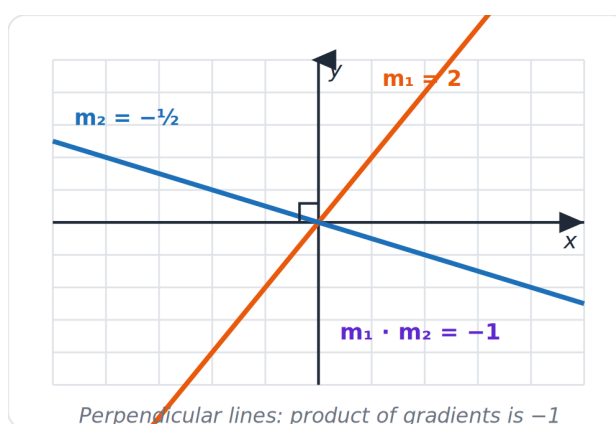
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EXERCISES (EASY → DIFFICULT)

1. What is true of the gradients of two parallel lines?
2. Is $y = 2x + 1$ parallel to $y = 2x - 5$?
3. Find the gradient of a line parallel to $y = 3x + 7$.
4. Find the equation of the line through $(0, 1)$ parallel to $y = 4x - 2$.
5. Find the equation of the line through $(2, 5)$ parallel to $y = 2x$.
6. Two railway tracks must stay parallel. One follows $y = \frac{1}{2}x + 3$. Find the line of a track through $(4, 1)$.
7. Are the lines through $(0,0)$ & $(2,4)$ and $(1,1)$ & $(3,5)$ parallel?
8. Find a if $y = ax + 2$ is parallel to $2x - y = 5$.
9. A line parallel to $y = -3x + 1$ passes through $(1, 2)$. Find its equation.
10. Show that the quadrilateral with sides $y = 2x$, $y = 2x + 4$ has two parallel sides.

4.5 Perpendicular lines



DEFINITIONS

Translation

Converting English sentences into algebra: "is" $\rightarrow =$, "more than" $\rightarrow +$, "twice" $\rightarrow 2 \times$, etc.

Let statement

A clear sentence saying what each variable stands for. Always start with this.

KEY FORMULAS

- Two lines are **perpendicular** $\Leftrightarrow m_1 \times m_2 = -1$.
- i.e. $m_2 = -1 / m_1$ (negative reciprocal)
- Horizontal line ($m = 0$) is perpendicular to vertical line (m undefined).

WORKED EXAMPLES

Example 1: Find the line through $(4, 0)$ perpendicular to $y = (1/3)x + 2$.

- Original gradient $m_1 = 1/3$, so $m_2 = -3$.

Negative reciprocal.

- Point-slope: $y - 0 = -3(x - 4) \rightarrow y = -3x + 12$.

Use the given point.



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**Applied example — A path across a river**

A straight river is $y = 3x$. A path must cross it at right angles through $(3, 9)$. Find the path.

► Perpendicular gradient = $-1/3$

Product of gradients = -1 .

► $y - 9 = -1/3(x - 3) \rightarrow y = -1/3x + 10$

Point-gradient form.

EXERCISES (EASY → DIFFICULT)

1. What is the product of the gradients of two perpendicular lines?
2. Find the gradient perpendicular to a line of gradient 2.
3. Find the gradient perpendicular to a line of gradient $-1/3$.
4. Is $y = 2x + 1$ perpendicular to $y = -1/2x + 4$?
5. Find the equation of the line through $(0, 0)$ perpendicular to $y = 2x$.
6. A path must cross a straight river $y = 3x$ at right angles, through $(3, 9)$. Find the path's equation.
7. Find the equation of the perpendicular bisector of $A(1, 2)$ and $B(5, 6)$.
8. Find k if $y = kx - 1$ is perpendicular to $y = 2x + 3$.
9. Show that the lines through $(0,1)&(2,5)$ and $(0,4)&(4,2)$ are perpendicular.
10. A roof beam of gradient $3/4$ meets a support at right angles. Find the support's gradient.

End of Chapter 4 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Find the distance between $(0, 0)$ and $(3, 4)$.
2. Find the equation of the line with gradient -1 through $(0, 4)$.
3. Find the inclination of a line with gradient $\sqrt{3}$.
4. Find the equation of the line through $(0, 1)$ parallel to $y = 4x - 2$.
5. Find the equation of the line through $(0, 0)$ perpendicular to $y = 2x$.
6. On a town map (km), a school is at $(2, 3)$ and a clinic at $(5, 7)$. How far apart are they?
7. A plumber charges a R350 call-out plus R250 per hour. Write the cost as a line.
8. Find the inclination of the line through $(1, 1)$ and $(4, 4)$.
9. A line parallel to $y = -3x + 1$ passes through $(1, 2)$. Find its equation.
10. A roof beam of gradient $3/4$ meets a support at right angles. Find the support's gradient.





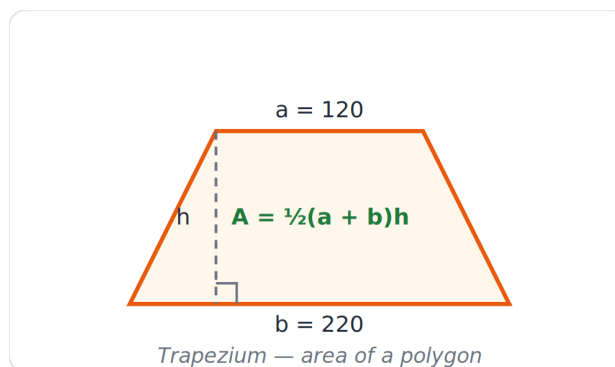
CHAPTER 7

Measurement

- 7.1 Area of polygons
- 7.2 Right prism / cylinder volume
- 7.3 Volume of a right cone
- 7.3 Volume of a sphere
- 7.4 Effect of multiplying a dimension by k

7.1 Area of polygon

Area of polygons



DEFINITIONS

Area	The space inside a 2-D shape, measured in square units.
Polygon	A closed flat shape with straight sides.

KEY FORMULAS

- **Rectangle:** $A = l \times b$
- **Triangle:** $A = \frac{1}{2} \times \text{base} \times \text{height}$
- **Trapezium:** $A = \frac{1}{2} \times (a + b) \times h$
- **Circle:** $A = \pi r^2$



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WORKED EXAMPLES

Example 1: Find the area of a triangle with base 10 cm and height 6 cm.

$$\triangleright A = \frac{1}{2} \times 10 \times 6 = 30 \text{ cm}^2.$$

Half base times height.

Applied example — Tiling a floor

A floor 6 m by 4 m is tiled with 0,5 m $\times$ 0,5 m tiles. How many tiles are needed?

$$\triangleright \text{Floor area} = 24 \text{ m}^2; \text{ tile area} = 0.25 \text{ m}^2$$

Area = length $\times$ width.

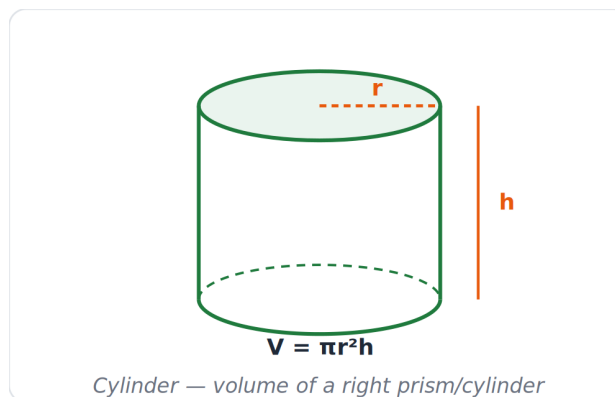
$$\triangleright 24 \div 0.25 = 96 \text{ tiles}$$

Divide.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. Find the area of a rectangle 8 cm by 5 cm.
2. Find the area of a triangle with base 10 cm and height 6 cm.
3. Find the area of a square of side 7 cm.
4. Find the area of a trapezium with parallel sides 6 cm and 10 cm, height 4 cm.
5. Find the area of a circle of radius 7 cm (use $\pi = 22/7$).
6. A rectangular plot is 25 m by 18 m. Find the area in m^2 .
7. A triangular sail has base 4 m and height 6 m. Find its area.
8. A circular tablecloth has radius 0,7 m ($\pi = 22/7$). Find its area.
9. A trapezoidal field has parallel sides 30 m and 50 m and width 20 m. Find the area.
10. A floor 6 m by 4 m is tiled with 0,5 m $\times$ 0,5 m tiles. How many tiles are needed?

7.2 Right prism / cylinder volume



DEFINITIONS

Right prism A solid with two identical parallel ends joined by rectangles.

Volume Amount of space a solid occupies = base area $\times$ height.

KEY FORMULAS



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- **Prism:** $V = \text{base area} \times \text{height}$
- **Cylinder:** $V = \pi r^2 \times h$
- Total surface area = sum of all faces
- **Cylinder SA:** $2\pi r^2 + 2\pi r h$

WORKED EXAMPLES

Example 1: A cylinder has radius 4 cm and height 10 cm. Find its volume.

► $V = \pi \times 4^2 \times 10 = 160\pi \approx 502.65 \text{ cm}^3$.

Substitute into $V = \pi r^2 h$.

Applied example — Litres in a tank

A rectangular tank is $2 \text{ m} \times 1,5 \text{ m} \times 1 \text{ m}$. How many litres does it hold?

► $V = 2 \times 1,5 \times 1 = 3 \text{ m}^3$

Volume of a rectangular prism.

► $3 \times 1000 = 3\,000 \text{ L}$

$1 \text{ m}^3 = 1000 \text{ L}$.

EXERCISES (EASY → DIFFICULT)

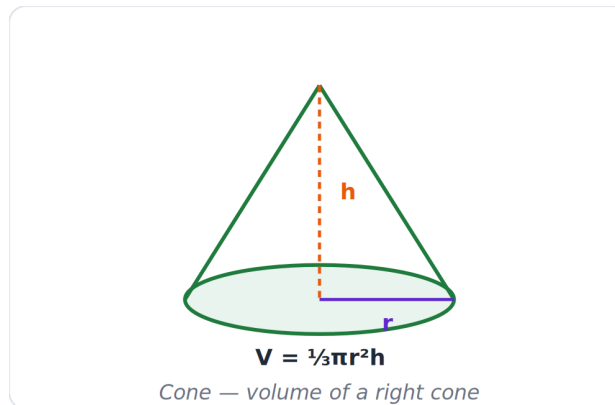
1. Find the volume of a box $4 \text{ cm} \times 3 \text{ cm} \times 5 \text{ cm}$.
2. Find the volume of a cube of side 5 cm.
3. Find the volume of a prism with base area 20 cm^2 and height 8 cm.
4. Find the volume of a cylinder with radius 7 cm, height 10 cm ($\pi = 22/7$).
5. Find the volume of a triangular prism: base triangle area 15 cm^2 , length 12 cm.
6. A rectangular water tank is $2 \text{ m} \times 1,5 \text{ m} \times 1 \text{ m}$. How many litres does it hold? ($1 \text{ m}^3 = 1000 \text{ L}$)
7. A cylindrical drum has radius 0,35 m and height 1 m ($\pi = 22/7$). Find its volume in m^3 , then litres.
8. A swimming pool $10 \text{ m} \times 4 \text{ m}$ has uniform depth 1,5 m. Find its volume.
9. A cylindrical tin ($\pi = 22/7$) of radius 7 cm holds $3\,080 \text{ cm}^3$. Find its height.
10. A concrete pillar is a cylinder of radius 0,2 m and height 3 m ($\pi \approx 3,14$). Find its volume (2 dp).

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7.3 Volume of right cone ($\pi=3$)

Volume of a right cone



DEFINITIONS

Cone A solid with a circular base tapering to a point (apex).

Cone volume $V = \frac{1}{3} \times \pi \times r^2 \times h$.

KEY FORMULAS

- **Cone:** $V = \frac{1}{3} \pi r^2 h$
- **Slant height:** $L = \sqrt{r^2 + h^2}$
- **Curved SA:** $\pi r L$. **Total SA:** $\pi r L + \pi r^2$.

WORKED EXAMPLES

Example 1: Cone with $r = 3$ m and $h = 9$ m. Find the volume.

► $V = \frac{1}{3} \times \pi \times 9 \times 9 = 27\pi \approx 84.82 \text{ m}^3$.

Substitute.

Applied example — An ice-cream cone

An ice-cream cone has radius 3 cm and height 12 cm ($\pi = 3$). How much fills it?

► $V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \cdot 3 \cdot 9 \cdot 12$

Cone volume formula.

► $= 108 \text{ cm}^3$

Evaluate.

EXERCISES (EASY → DIFFICULT)

1. Find the volume of a cone with radius 5 cm, height 6 cm ($\pi = 3$).
2. Find the volume of a cone with radius 2 cm, height 9 cm ($\pi = 3$).
3. Find the volume of a cone with radius 10 cm, height 3 cm ($\pi = 3$).
4. A cone has radius 4 cm and height 6 cm ($\pi = 3$). Find its volume.
5. A cone has radius 3 cm and height 10 cm ($\pi = 3$). Find its volume.
6. An ice-cream cone has radius 3 cm and height 12 cm ($\pi = 3$). How much ice cream fills it?
7. A conical pile of sand has radius 2 m and height 1,5 m ($\pi = 3$). Find its volume.
8. A conical funnel of radius 5 cm and height 9 cm ($\pi = 3$) — find its capacity.
9. A cone ($\pi = 3$) of radius 6 cm has volume 360 cm^3 . Find its height.
10. A party hat is a cone of radius 7 cm and height 24 cm ($\pi = 3$). Find its volume.

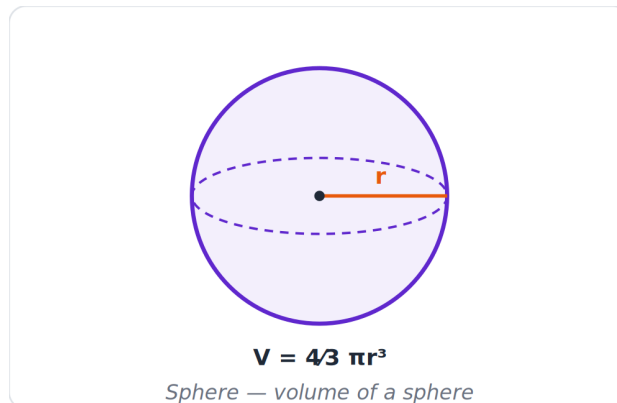


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7.3 Volume of sphere

Volume of a sphere



DEFINITIONS

Sphere A perfectly round solid; every surface point is equidistant from the centre.

Sphere volume $V = \frac{4}{3} \times \pi \times r^3$.

KEY FORMULAS

- **Sphere:** $V = \frac{4}{3} \pi r^3$
- **Sphere SA:** $4 \pi r^2$

WORKED EXAMPLES

Example 1: Find the volume of a sphere with radius 5 cm.

► $V = \frac{4}{3} \pi \times 125 = \frac{500}{3} \pi \approx 523.6 \text{ cm}^3$.

Substitute $r = 5$.

Applied example — A spherical tank

A spherical tank has radius 0,5 m ($\pi = 3$). How many litres does it hold?

► $V = \frac{4}{3} \cdot \pi \cdot r^3 = \frac{4}{3} \cdot 3 \cdot 0.125$

Sphere volume formula.

► $= 0.5 \text{ m}^3 = 500 \text{ L}$

Convert m^3 to litres.

EXERCISES (EASY → DIFFICULT)

1. Find the volume of a sphere of radius 3 cm ($\pi = 3$).
2. Find the volume of a sphere of radius 6 cm ($\pi = 3$).
3. Find the volume of a sphere of radius 1 cm ($\pi = 3$).
4. Find the volume of a sphere of radius 5 cm ($\pi = 3$).
5. Find the volume of a sphere of radius 2 cm ($\pi = 3$).
6. A spherical ball has radius 3 cm ($\pi = 3$). Find its volume.
7. A soccer ball has radius 11 cm ($\pi \approx 3,14$). Find its volume (nearest cm^3).
8. A spherical water tank has radius 0,5 m ($\pi = 3$). Find its volume in m^3 , then litres.
9. A marble has radius 1 cm; a box holds 4 cm^3 of plastic. How many marbles ($\pi = 3$)?



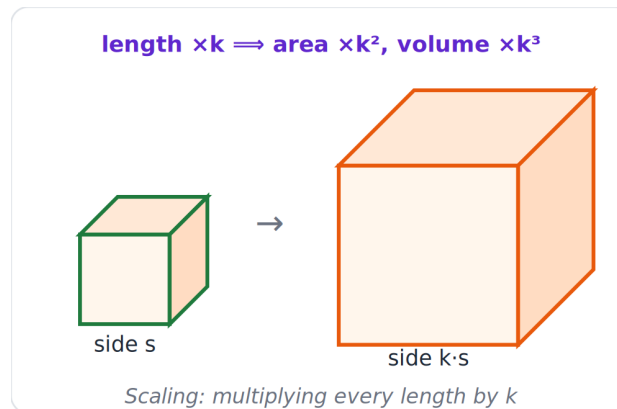
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10. A sphere ($\pi = 3$) has volume 256 cm^3 . Find its radius.

7.4 Multiplying a dimension by k

Effect of multiplying a dimension by k



DEFINITIONS

Mid-point

The point that divides a line segment into two EQUAL halves.

Mid-point theorem

In any triangle, the line joining the mid-points of two sides is PARALLEL to the third side and HALF its length.

KEY FORMULAS

- If ONE dimension is scaled by k: length $\times k$, area $\times k$, volume $\times k$.
- If ALL dimensions scaled by k: length $\times k$, area $\times k^2$, volume $\times k^3$.
- Scaling height alone: only volume $\times k$.
- Scaling height AND radius: $V \times k^3$.

WORKED EXAMPLES

Example 1: A cylinder's radius is tripled. What happens to its volume?

- $V = \pi r^2 h$. Radius gets a $\times 3$, but r is squared.

So r^2 gets $\times 9$.

- V becomes $9 \times$ larger.

Height untouched.

Applied example — Scaling a tank

A water tank's dimensions are all multiplied by k, changing 30 cm^3 to 810 cm^3 . Find k.

- Volume scales by k^3 : $30k^3 = 810$

Linear factor cubed for volume.

- $k^3 = 27 \rightarrow k = 3$

Solve.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. A square's side is doubled. By what factor does its area change?
2. A cube's side is doubled. By what factor does its volume change?
3. A length is multiplied by 3. By what factor does the area change?
4. A length is multiplied by 3. By what factor does the volume change?



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5. A length is halved. By what factor does the volume change?
6. A cylinder's radius and height are both doubled. By what factor does its volume change?
7. A model car is built at scale 1:10. By what factor is the real surface area larger?
8. A water tank's dimensions are all increased by 50% ($\times 1,5$). By what factor does it hold more?
9. A photo of area 12 cm^2 is enlarged so each side $\times 4$. Find the new area.
10. A box of volume 30 cm^3 has every dimension multiplied by k , giving 810 cm^3 . Find k .

End of Chapter 7 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Find the area of a rectangle 8 cm by 5 cm.
2. Find the volume of a cube of side 5 cm.
3. Find the volume of a cone with radius 10 cm, height 3 cm ($\pi = 3$).
4. Find the volume of a sphere of radius 5 cm ($\pi = 3$).
5. A length is halved. By what factor does the volume change?
6. A rectangular plot is 25 m by 18 m. Find the area in m^2 .
7. A cylindrical drum has radius 0,35 m and height 1 m ($\pi = 22/7$). Find its volume in m^3 , then litres.
8. A conical funnel of radius 5 cm and height 9 cm ($\pi = 3$) — find its capacity.
9. A marble has radius 1 cm; a box holds 4 cm^3 of plastic. How many marbles ($\pi = 3$)?
10. A box of volume 30 cm^3 has every dimension multiplied by k , giving 810 cm^3 . Find k .

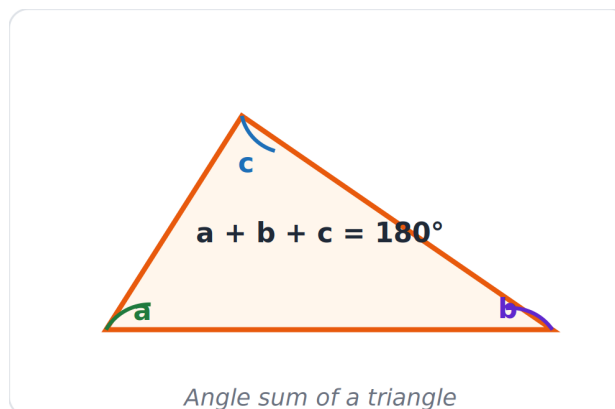


CHAPTER 8

Euclidean geometry — circle theorems

- 8.1 8.1 Revision: angle sum of triangle
- 8.2 Tangent perpendicular to radius
- 8.2 Perpendicular from centre bisects chord
- 8.2 Angle at centre = $2 \times$ angle at circumference
- 8.2 Angle in a semi-circle
- 8.2 Angles in the same segment
- 8.2 Cyclic quadrilateral
- 8.2 Tangent-chord theorem
- 8.2 Two tangents from external point

8.1 Revision: angle sum of triangle



DEFINITIONS

Angle sum of a triangle The three interior angles of any triangle add to 180° .

Exterior angle Equals the sum of the two opposite interior angles.



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WORKED EXAMPLES

Example 1: Worked example: Two angles 59° , 58° . Find the third.

- ▶ Angle sum = 180° .
- ▶ $180 - 59 - 58 = 63$.
- ▶ Answer: 63.

Applied example — A roof gable

A roof gable is an isosceles triangle with apex 50° . Find each base angle.

- ▶ Base angles equal; sum is 180°

Isosceles triangle.

- ▶ $(180 - 50)/2 = 65^\circ$

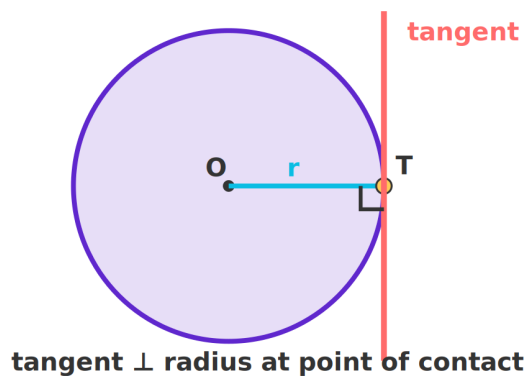
Evaluate.

EXERCISES (EASY → DIFFICULT)

1. Two angles of a triangle are 70° and 60° . Find the third.
2. Two angles are 90° and 35° . Find the third.
3. Each angle of an equilateral triangle is...
4. The angles are x , $2x$ and $3x$. Find x .
5. An exterior angle of a triangle is 110° ; one remote interior angle is 45° . Find the other.
6. A triangular road sign is isosceles with apex 50° . Find each base angle.
7. A roof gable triangle has angles in the ratio 2:3:4. Find the largest angle.
8. In $\triangle ABC$, $\angle A = \angle B$ and $\angle C = 70^\circ$. Find $\angle A$.
9. A triangle has angles $(x + 10)$, $(2x)$, $(x + 30)$. Find x .
10. A bracing triangle is right-angled and isosceles. Find its three angles.

8.2 Tangent $\perp$ radius

Tangent perpendicular to radius

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Tangent	A straight line that touches the circle at EXACTLY ONE point.
Radius	A straight line from the centre of the circle to any point on the circle. All radii of one circle are equal.
Point of contact	The single point where the tangent touches the circle.
Perpendicular	Two lines that meet at exactly 90° (a right angle).

KEY FORMULAS

- **Theorem:** A tangent to a circle is PERPENDICULAR to the radius drawn to the point of contact.
- $\angle$ between tangent and radius = 90° .
- **Converse:** If a line through a point on the circle is perpendicular to the radius there, it is a tangent.

Applied example — Tangent length

A tangent PT touches a circle, centre O, radius 6. If $OP = 10$, find PT.

- $\angle OTP = 90^\circ$ (tangent $\perp$ radius)

Right angle at the contact point.

- $PT = \sqrt{(10^2 - 6^2)} = \sqrt{64} = 8$

Pythagoras.

EXERCISES (EASY → DIFFICULT)

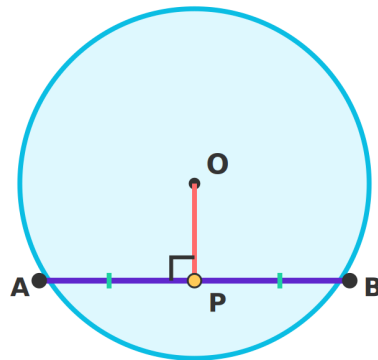
1. What is the angle between a tangent and the radius at the point of contact?
2. A tangent touches a circle in how many points?
3. OT is a radius and the tangent at T makes angle x with OT. Find x .
4. A radius to the point of contact has gradient 2. Find the tangent's gradient.
5. $OT = 5$, and a tangent from external point P has $PT = 12$. Find OP.
6. A bicycle wheel touches the road. What angle does the road (tangent) make with the spoke (radius) at that point?
7. A tangent at T meets radius OT. If $\angle OTP = 90^\circ$ and $\angle TOP = 40^\circ$, find $\angle OPT$.
8. A circular pond of radius 8 m has a straight path tangent to it. A post P is 15 m from the centre along the path direction; find the tangent length from P.
9. Tangent PT touches a circle, centre O, radius 6. If $OP = 10$, find PT.
10. Why must the tangent be perpendicular to the radius at the contact point?



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8.2 Perpendicular from centre bisects chord



$\perp$ from centre bisects chord: $AP = PB$

DEFINITIONS

Chord	A straight line segment whose two endpoints both lie on the circle.
Perpendicular bisector	A line that crosses another line at 90° AND cuts it into two equal halves.
Centre	The single point inside the circle that is equidistant from every point on the circle.
Bisect	To divide something exactly into two equal parts.

KEY FORMULAS

- **Theorem:** Two tangents drawn from the SAME external point to a circle are EQUAL in length.
- If TA and TB are tangents from external point T, then $TA = TB$.
- The line from the external point to the centre BISECTS the angle between the tangents.

Applied example — A bridge arch

A bridge arch is a circular arc of radius 13 m. A horizontal beam sits 5 m below the centre. Find its length.

- Perpendicular from centre bisects the chord

So half the beam = $\sqrt{(13^2 - 5^2)}$.

► $= \sqrt{144} = 12 \rightarrow$ beam = 24 m

Double the half-length.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. A perpendicular from the centre to a chord does what to the chord?
2. A chord is 16 cm long. The perpendicular from the centre meets it at distance...from each end.
3. Radius 10, the perpendicular distance from centre to a chord is 6. Find half the chord.
4. Radius 13, chord 24. Find the distance from the centre to the chord.
5. Radius 5, chord 8. Find the distance from the centre to the chord.
6. A circular window of radius 25 cm has a horizontal glazing bar (chord) 14 cm below the centre. Find its length.

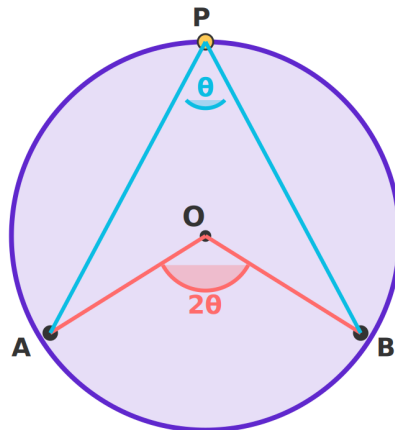


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7. A bridge arch is a circular arc, radius 13 m. A horizontal beam (chord) sits 5 m below the centre. Find the beam length.
8. Radius 17, chord 30. Find the distance from the centre to the chord.
9. A chord of length 12 lies 8 from the centre. Find the radius.
10. Two parallel chords of a circle (radius 10) are 12 and 16 long, on the same side. Find the distance between them.

8.2 Angle at centre = 2 × angle at circumference



$$\angle \text{ at centre} = 2 \times \angle \text{ at circumference}$$

DEFINITIONS

Central angle	An angle whose VERTEX is at the CENTRE of the circle.
Inscribed angle	An angle whose VERTEX sits ON the circle (also called a circumference angle).
Arc	A continuous portion of the circle's edge between two points.
Subtend	An angle is "subtended by" an arc or chord — the arc/chord lies opposite the angle's vertex.

KEY FORMULAS

- **Theorem:** The angle subtended by an arc at the CENTRE is TWICE the angle subtended at the circumference.
- $\angle \text{ at centre} = 2 \times \angle \text{ at circumference}$ (when both stand on the same arc).
- **Special case:** the angle in a SEMICIRCLE is 90° (diameter subtends 180° at the centre).

Applied example — A Ferris wheel

Two cabins and the hub make 120° at the hub. What angle do they make at a rim point on the same arc?

- Inscribed angle = $\frac{1}{2} \times \text{central angle}$
 $\text{Angle at centre} = 2 \times \text{angle at circumference.}$
- $= \frac{1}{2} \times 120^\circ = 60^\circ$
Evaluate.



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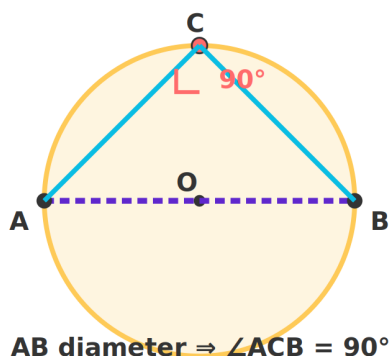
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EXERCISES (EASY → DIFFICULT)

1. The angle at the centre is what multiple of the angle at the circumference (same arc)?
2. Angle at centre = 80° . Find the angle at the circumference (same arc).
3. Angle at circumference = 35° . Find the angle at the centre.
4. Angle at centre = 140° . Find the angle at the circumference.
5. Angle at circumference = 50° . Find the reflex... no — find the central angle.
6. On a Ferris wheel, two cabins and the hub form an angle of 120° at the hub. What angle do they make at a point on the rim (same arc)?
7. A central angle of 100° subtends an arc. Find the inscribed angle on that arc.
8. The inscribed angle is $2x$ and the central angle on the same arc is 100° . Find x .
9. Central angle = $x + 20$, inscribed angle = $x - 10$ (same arc). Find x .
10. Why is the central angle double the inscribed angle? (one-line reason)

8.2 Angle in semi-circle = 90°

Angle in a semi-circle



DEFINITIONS

Diameter	A chord that passes through the centre of the circle. It is the LONGEST possible chord and equals $2 \times$ radius.
Semicircle	Half of a circle, cut off by a diameter.
Right angle	An angle of exactly 90° .
Inscribed angle	An angle whose vertex sits on the circle.

KEY FORMULAS

- **Theorem:** Angles subtended by the SAME arc at the circumference are EQUAL.
- If P and Q are both on the major arc above chord AB, then $\angle APB = \angle AQB$.
- Follows directly from 8.2.c — both equal half the centre angle $\angle AOB$.



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**Applied example — A semicircular arch**

AB is a diameter of length 13. Point C on the circle has $AC = 5$. Find BC.

► $\angle ACB = 90^\circ$ (angle in a semicircle)

Right angle.

► $BC = \sqrt{13^2 - 5^2} = 12$

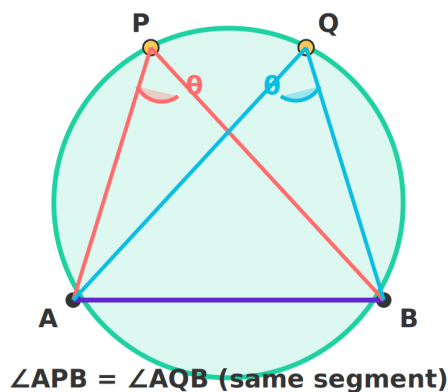
Pythagoras.

EXERCISES (EASY → DIFFICULT)

1. The angle in a semicircle equals...
2. AB is a diameter and C is on the circle. Find $\angle ACB$.
3. In a semicircle, one acute angle of the triangle is 35° . Find the third angle.
4. AB is a diameter, $\angle BAC = 40^\circ$. Find $\angle ABC$.
5. A right angle is inscribed in a circle. What must the hypotenuse be?
6. A Ferris wheel has a horizontal diameter. A cabin on the rim is joined to both ends. What angle is at the cabin?
7. AB is a diameter of length 10. C is on the circle with $AC = 6$. Find BC.
8. AB is a diameter, $\angle ABC = 25^\circ$. Find $\angle ACB$ and $\angle BAC$.
9. A semicircular arch spans 10 m (diameter). A strut from one base to a rim point makes the apex angle...
10. AB is a diameter, C on the circle, $AC = 5$, $BC = 12$. Find the diameter AB.

8.2 Angles in same segment are equal

Angles in the same segment

**DEFINITIONS**

Segment	The region of a circle cut off by a chord. A chord divides the disk into two segments — major and minor.
Same segment	Two points lie "in the same segment" if they are both on the SAME side of the chord that creates the segment.
Subtended	An angle is "subtended by chord AB" when its two sides terminate at A and B.
Major / minor	Major = larger; Minor = smaller. Used for both arcs and segments.

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KEY FORMULAS

- **Theorem:** Opposite angles of a CYCLIC QUADRILATERAL are SUPPLEMENTARY (add to 180°).
- $\angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180^\circ$.
- **Converse:** if opposite angles of a quadrilateral sum to 180° , the quadrilateral is cyclic.

Applied example — Two spectators

Two spectators on a circular arena view the same goal arc; one sees it at 40° . What does the other see?

- Angles in the same segment are equal

Same arc, same side.

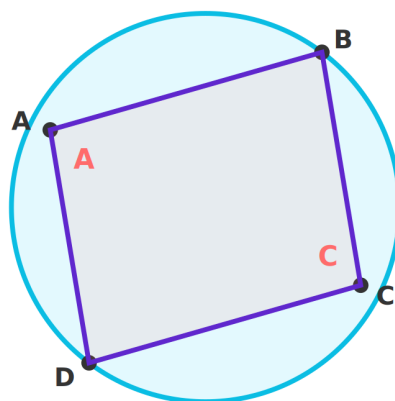
- The other also sees 40°

Equal angles.

EXERCISES (EASY → DIFFICULT)

- Angles in the same segment are...
- Two angles in the same segment: one is 38° . Find the other.
- $\angle ACB$ and $\angle ADB$ stand on the same arc AB. If $\angle ACB = 52^\circ$, find $\angle ADB$.
- Same segment: angles are $2x$ and 50° . Find x .
- Same segment: angles are $(x + 10)$ and $(2x - 20)$. Find x .
- Two spectators on a circular arena rim view the same goal arc. One sees it at 40° . What does the other see?
- $\angle P$ and $\angle Q$ are in the same segment; $\angle P = 3x$ and $\angle Q = x + 30$. Find $\angle P$.
- On arc AB, $\angle ACB = 35^\circ$. The central angle $\angle AOB$ on the same arc is...
- Two angles subtend the same chord from the same side; one is $(4x - 10)$, the other $(2x + 30)$. Find x .
- Why are angles in the same segment equal? (one line)

8.2 Cyclic quadrilateral — opposite angles supplementary



$$\angle A + \angle C = 180^\circ \text{ (opposite angles supp.)}$$



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DEFINITIONS

Cyclic quadrilateral	A four-sided polygon (quadrilateral) whose four vertices ALL lie on a single circle.
Supplementary	Two angles are supplementary if they add up to 180° .
Opposite angles	In quadrilateral ABCD, A and C are opposite angles; B and D are opposite angles.
Vertices	The corner points of a polygon. A quadrilateral has 4 vertices.

KEY FORMULAS

- **Tan-chord theorem:** the angle between a TANGENT and a CHORD equals the angle in the ALTERNATE SEGMENT.
- If TP is tangent at P and PQ is a chord, then $\angle TPQ = \angle PRQ$ for any R on the major arc.

Applied example — A round window

A round window holds a cyclic quadrilateral ABCD with $\angle A = 88^\circ$. Find $\angle C$.

► $\angle A + \angle C = 180^\circ$

Opposite angles of a cyclic quad are supplementary.

► $\angle C = 92^\circ$

Subtract.

EXERCISES (EASY → DIFFICULT)

1. Opposite angles of a cyclic quadrilateral are...
2. One angle of a cyclic quad is 80° . Find the opposite angle.
3. One angle of a cyclic quad is 95° . Find the opposite angle.
4. Cyclic quad angles are x and 110° (opposite). Find x .
5. The exterior angle of a cyclic quad is 75° . Find the interior opposite angle.
6. A round window holds a cyclic quad ABCD with $\angle A = 88^\circ$. Find $\angle C$.
7. Three angles of a cyclic quad are 70° , 100° , 110° . Find the fourth.
8. Cyclic quad: $\angle A = 2x$, $\angle C = x + 30$ (opposite). Find x .
9. Cyclic quad ABCD: $\angle B = 3x$ and $\angle D = x + 20$ (opposite). Find $\angle B$.
10. Show that if one pair of opposite angles of a quadrilateral sums to 180° , it is cyclic.

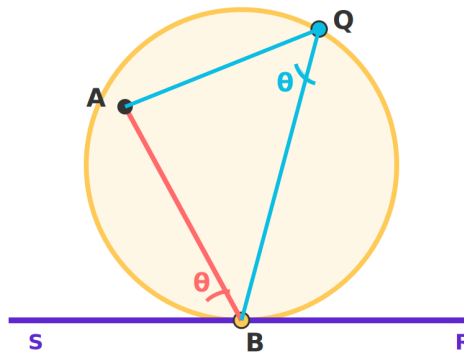


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8.2 Tangent-chord theorem (alternate segment)

tangent-chord = alternate segment angle



DEFINITIONS

Tangent	A line that touches the circle at exactly one point.
Chord	A line segment joining two points on the circle.
Alternate segment	When a tangent and a chord meet at a point on the circle, they form two angles. The "alternate segment" is the segment of the circle on the OPPOSITE side of the chord from the angle you are looking at.
Subtended in segment	An angle is "subtended by the chord in the alternate segment" when its vertex sits in that segment and its sides terminate at the chord's endpoints.

KEY FORMULAS

- **Converse of tan-chord:** if a line through the endpoint of a chord makes an angle equal to the angle in the alternate segment, that line is a TANGENT.
- **Use:** pick the angle you can measure, then conclude tangency.

Applied example — A circular logo

On a circular logo, a straight edge (tangent) meets a chord at 55° . Find the inscribed angle it equals.

- Tangent-chord angle = angle in the alternate segment
Alternate segment theorem.
- $= 55^\circ$
Equal angles.

EXERCISES (EASY → DIFFICULT)

1. State the tangent-chord (alternate segment) theorem.
2. A tangent-chord angle is 50° . Find the angle in the alternate segment.
3. The angle in the alternate segment is 65° . Find the tangent-chord angle.
4. Tangent-chord angle $= 2x$ and alternate-segment angle $= 70^\circ$. Find x .
5. A tangent-chord angle is 90° . What does the chord become?
6. A tangent at T and chord TA make 48° . The inscribed angle TBA in the alternate segment is...



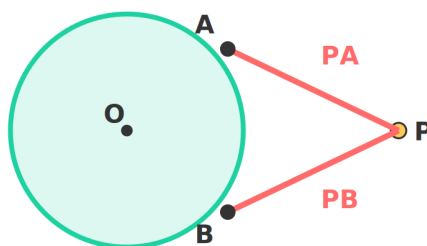
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7. Tangent-chord angle = $(x + 10)$; alternate angle = $(2x - 30)$. Find x .
8. A circular logo: a straight edge (tangent) meets a chord at 55° . Find the inscribed angle it equals.
9. Two tangent-chord angles on the two sides of a chord are 50° and 60° . Find the third angle of the inscribed triangle on that chord.
10. Why does the tangent-chord angle equal the alternate-segment angle? (one-line idea)

8.2 Two tangents from an external point

Two tangents from external point



$PA = PB$ (tangents from same point)

DEFINITIONS

External point	A point that lies OUTSIDE the circle (further from the centre than the radius).
Tangent length	The distance from the external point along the tangent line to the point of contact with the circle.
Congruent	Exactly the same shape AND size. Congruent triangles have corresponding sides and angles equal.
Bisects	Cuts into two equal halves. The line from an external point to the centre bisects the angle between the two tangents.

KEY FORMULAS

- **Intersecting chords:** if two chords AB and CD cross at point X inside the circle, then $AX \cdot XB = CX \cdot XD$.
- **Secant-tangent:** if a tangent and a secant meet outside the circle, $(\text{tangent})^2 = (\text{whole secant}) \times (\text{external segment})$.
- Both come from similar triangles formed by the intersecting lines.



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Applied example — Two tangents from a point

From a point P, two straight edges just touch a circular fountain. One is 7 m. Find OP if the radius is also known? Find the other tangent.

- Tangents from an external point are equal

$$PA = PB.$$

- The other tangent is also 7 m

Equal lengths.

EXERCISES (EASY → DIFFICULT)

1. Two tangents are drawn from an external point. What is true of their lengths?
2. From P, two tangents touch a circle. One is 9 cm. Find the other.
3. Tangent length from P is 12; the line PO bisects the angle between the tangents. True or false?
4. PA and PB are tangents with $\angle APB = 50^\circ$. Find $\angle APO$.
5. PA = PB tangents; $\angle PAB = 65^\circ$. Find $\angle APB$.
6. From a point P outside a circular fountain, two straight edges just touch the rim. One edge measures 7 m; how long is the other?
7. PA, PB tangents, radius 5, OP = 13. Find the tangent length PA.
8. PA, PB tangents; $\angle AOB$ (at centre) = 120° . Find $\angle APB$.
9. Two tangents from P meet the circle at A and B, and $\angle APB = 70^\circ$. Find $\angle OAP + \angle OBP$.
10. From P, tangents touch the circle at A and B. If PA = PB = 8 and AB = 8, what kind of triangle is PAB?

End of Chapter 8 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Two angles of a triangle are 70° and 60° . Find the third.
2. A tangent touches a circle in how many points?
3. Radius 10, the perpendicular distance from centre to a chord is 6. Find half the chord.
4. Angle at centre = 140° . Find the angle at the circumference.
5. A right angle is inscribed in a circle. What must the hypotenuse be?
6. Two spectators on a circular arena rim view the same goal arc. One sees it at 40° . What does the other see?
7. Three angles of a cyclic quad are 70° , 100° , 110° . Find the fourth.
8. A circular logo: a straight edge (tangent) meets a chord at 55° . Find the inscribed angle it equals.
9. Two tangents from P meet the circle at A and B, and $\angle APB = 70^\circ$. Find $\angle OAP + \angle OBP$.
10. A bracing triangle is right-angled and isosceles. Find its three angles.



**CHAPTER 9**

Finance, growth and decay

- 9.1** Revision: compound interest
- 9.2** Simple (straight-line) depreciation
- 9.2** Compound (reducing-balance) depreciation
- 9.3** Timelines with multiple interest rates
- 9.4** Nominal vs effective interest

9.1 Revision: compound interest

DEFINITIONS

Compound interest Interest on the original amount and on accumulated interest.

Formula $A = P(1 + i)^n$.

KEY FORMULAS

- **Compound interest:** $A = P(1 + i)^n$
- P = principal, i = rate per period (as decimal), n = number of periods
- **Simple interest:** $A = P(1 + in)$
- Compound > Simple over long time periods.

WORKED EXAMPLES

Example 1: R 1 000 invested at 8% compounded annually for 5 years. Find the final amount.

► $A = 1000 \times (1.08)^5 \approx 1469.33$.

Substitute and use a calculator.

Applied example — Compound growth

Thabo invests R12 000 at 8,5% p.a. compounded annually. Find its value after 6 years.

► $A = P(1 + i)^n = 12000(1.085)^6$

Compound interest formula.

► $A \approx \text{R}19,577.61$

Evaluate.

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EXERCISES (EASY → DIFFICULT)

1. Find the value of R5 000 after 3 years at 8% p.a. compound interest.
2. Find the value of R10 000 after 2 years at 10% p.a. compounded annually.
3. Find the value of R2 000 after 4 years at 6% p.a. compounded annually.
4. Find the value of R8 000 after 5 years at 9% p.a. compounded annually.
5. Find the interest earned on R4 000 after 2 years at 7% p.a. compounded annually.
6. Thabo invests R12 000 at 8,5% p.a. compounded annually. Find its value after 6 years.
7. Naledi invests R15 000 at 9% p.a. compounded monthly for 3 years. Find the value.
8. A R20 000 investment grows at 7,2% p.a. compounded quarterly for 5 years. Find its value.
9. How long until R5 000 doubles at 10% p.a. compounded annually?
10. At what annual rate (compounded yearly) does R8 000 grow to R12 000 in 4 years?

9.2 Simple depreciation (straight line)

Simple (straight-line) depreciation

DEFINITIONS

Simple depreciation	Value lost by a fixed amount each year (straight-line).
Formula	$A = P(1 - in)$.

KEY FORMULAS

- **Straight-line:** $A = P(1 - in)$
- Same amount is subtracted each year.
- Asset reaches scrap value after a fixed number of years.

WORKED EXAMPLES

Example 1: A car worth R 200 000 depreciates straight-line at 12% per year. Value after 4 years?

$$\triangleright A = 200\,000 \times (1 - 0.12 \times 4) = 200\,000 \times 0.52 = R\,104\,000.$$

Direct substitution.

Applied example — Straight-line depreciation

A R250 000 bakkie depreciates 12,5% p.a. on the straight-line method. Find its value after 4 years.

$$\triangleright A = P(1 - i \cdot n) = 250\,000(1 - 0.125 \cdot 4)$$

Straight-line (simple) depreciation.

$$\triangleright = R125\,000.00$$

Evaluate.

EXERCISES (EASY → DIFFICULT)

1. A R10 000 asset depreciates 10% p.a. on the straight-line method. Find its value after 1 year.
2. Same asset on straight-line: find its value after 3 years.
3. A R20 000 vehicle depreciates 15% p.a. (straight-line). Value after 2 years?
4. A R6 000 laptop depreciates 20% p.a. (straight-line). Value after 4 years?
5. On the straight-line method, what fraction of the original value is lost each year at 12% p.a.?



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6. Lerato's R250 000 bakkie depreciates 12,5% p.a. (straight-line). Find its value after 4 years.
7. A R30 000 machine depreciates straight-line at 16% p.a. After how many years is it worthless?
8. A printer cost R8 000 and is worth R3 200 after 4 years (straight-line). Find the annual rate.
9. A R18 000 asset depreciates straight-line to R6 000 over 5 years. Find the annual depreciation in rand.
10. A R40 000 vehicle loses R5 000 of value each year (straight-line). What is the annual rate?

9.2 Compound depreciation (reducing balance)

Compound (reducing-balance) depreciation

DEFINITIONS

Compound depreciation	Value lost by a fixed percentage each year (reducing balance).
Formula	$A = P(1 - i)^n$.

KEY FORMULAS

- **Reducing balance:** $A = P(1 - i)^n$
- Each year, the asset loses a PERCENTAGE of its *current* value.
- Depreciation amount shrinks each year.

WORKED EXAMPLES

Example 1: A laptop worth R 12 000 depreciates 15% per year on reducing balance. Value after 3 years?

$$\triangleright A = 12\,000 \times (0.85)^3 = 12\,000 \times 0.6141 \approx R\,7\,370.$$

Substitute.

Applied example — Reducing-balance depreciation

A R250 000 bakkie depreciates 18% p.a. on the reducing-balance method. Find its value after 5 years.

$$\triangleright A = P(1 - i)^n = 250\,000(0.82)^5$$

Reducing-balance depreciation.

$$\triangleright \approx R92\,684.96$$

Evaluate.

EXERCISES (EASY → DIFFICULT)

1. A R10 000 asset depreciates 10% p.a. on the reducing-balance method. Value after 1 year?
2. Same asset, reducing-balance: value after 3 years?
3. A R20 000 car depreciates 15% p.a. (reducing-balance). Value after 2 years?
4. A R6 000 phone depreciates 20% p.a. (reducing-balance). Value after 3 years?
5. Reducing balance vs straight-line: which leaves more value in later years?
6. Sipho's R250 000 bakkie depreciates 18% p.a. (reducing-balance). Value after 5 years?
7. A R35 000 machine depreciates 12% p.a. (reducing-balance). Find its value after 4 years.
8. A laptop worth R12 000 is worth R6 144 after 3 years (reducing-balance). Find the annual rate.



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9. A R50 000 vehicle depreciates 16% p.a. (reducing-balance). After how many years is it worth under R20 000?
10. A tractor halves in value in 4 years (reducing-balance). Find the annual depreciation rate.

9.3 Timelines (multiple rates)

Timelines with multiple interest rates

DEFINITIONS

Compound interest

Interest is added to the principal at the end of each period, then EARNs INTEREST itself in the next period — interest on interest.

Compounding period

How often interest is added: yearly, monthly, etc. More frequent = faster growth.

KEY FORMULAS

- Split the timeline at each rate change.
- Compute A at the end of each segment, then use that A as P for the next.
- Be careful with compounding period (annual, monthly, etc.).

WORKED EXAMPLES

Example 1: R 5 000 invested for 3 years at 6%, then for 4 years at 9% (both compounded annually). Final amount?

- After 3 yrs: $5\,000 \times 1.06^3 \approx 5\,955.08$.

First segment.

- Then: $5\,955.08 \times 1.09^4 \approx 8\,408.65$.

Second segment.

Applied example — Two interest rates

R5 000 earns 8% p.a. for 2 years, then 10% p.a. for 3 years (compounded annually). Find the final value.

- $A = 5000(1.08)^2(1.10)^3$

Apply each rate over its period on a timeline.

- $\approx R7,762.39$

Evaluate.

EXERCISES (EASY → DIFFICULT)

1. R5 000 earns 8% p.a. for 2 years, then 10% p.a. for 3 years (all compounded annually). Find the final value.
2. R8 000 grows at 9% p.a. for 4 years. Find its value.
3. R10 000 earns 6% p.a. for 3 years then 7% p.a. for 2 years. Find the value.
4. A R2 000 deposit grows at 12% p.a. compounded monthly for 2 years. Find the value.
5. On a timeline, money is moved to the end using which operation per period?
6. Thabo deposits R6 000 now and R4 000 in 2 years; the account earns 9% p.a. compounded annually. Find the total after 5 years.
7. R12 000 earns 10% p.a. for the first 3 years, then the rate drops to 8% p.a. for 2 years. Find the value after 5 years.



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8. Naledi withdraws R3 000 from an account after it has grown. She invests R10 000 at 9% p.a.; after 2 years she withdraws R3 000, then leaves it 3 more years. Find the final value.
9. R15 000 grows at 7% p.a. compounded half-yearly for 4 years. Find the value.
10. R20 000 is invested. After 5 years at 8% p.a. compounded quarterly, find the value.

9.4 Nominal vs effective interest

DEFINITIONS

Linear growth	Equal absolute increase per period — gives a STRAIGHT-LINE graph (simple interest).
Exponential growth	Equal PERCENTAGE increase per period — gives a CURVED graph getting steeper (compound interest).

KEY FORMULAS

- **Nominal rate** = quoted annual rate (e.g. "12% per annum compounded monthly").
- **Effective rate** = actual annual growth experienced.
- **Conversion:** $(1 + i_{\text{eff}}) = (1 + i_{\text{nom}} / m)^m$ where m = compoundings per year.

WORKED EXAMPLES

Example 1: 15% p.a. compounded quarterly. Find the effective annual rate.

► $(1 + 0.15/4)^4 = 1.0375^4 \approx 1.1587$.

4 quarters per year.

► Effective rate $\approx 15.87\%$.

Slightly higher than nominal.

Applied example — Comparing two banks

Bank A offers 9% compounded monthly; Bank B offers 9,3% compounded annually. Which is better?

► Bank A effective = $(1 + 0.09/12)^{12} - 1 \approx 9.381\%$

Convert nominal to effective.

► $9.38\% > 9.3\% \rightarrow$ choose Bank A

Compare like with like.

EXERCISES (EASY → DIFFICULT)

1. Convert 12% p.a. compounded monthly to a monthly rate.
2. Find the effective annual rate of 12% p.a. compounded monthly.
3. Find the effective annual rate of 10% p.a. compounded quarterly.
4. Find the effective annual rate of 8% p.a. compounded half-yearly.
5. Which is larger: nominal or effective rate, when compounding more than once a year?
6. Bank A offers 9% compounded monthly; Bank B offers 9,3% compounded annually. Which is better?
7. Find the effective annual rate of 15% p.a. compounded monthly.
8. A nominal rate compounded quarterly gives effective 8,24%. Estimate the nominal rate.
9. Lerato compares 6% compounded monthly with 6,1% compounded annually. Which earns more?
10. Find the effective annual rate of 18% p.a. compounded monthly.



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End of Chapter 9 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Find the value of R5 000 after 3 years at 8% p.a. compound interest.
2. Same asset on straight-line: find its value after 3 years.
3. A R20 000 car depreciates 15% p.a. (reducing-balance). Value after 2 years?
4. A R2 000 deposit grows at 12% p.a. compounded monthly for 2 years. Find the value.
5. Which is larger: nominal or effective rate, when compounding more than once a year?
6. Thabo invests R12 000 at 8,5% p.a. compounded annually. Find its value after 6 years.
7. A R30 000 machine depreciates straight-line at 16% p.a. After how many years is it worthless?
8. A laptop worth R12 000 is worth R6 144 after 3 years (reducing-balance). Find the annual rate.
9. R15 000 grows at 7% p.a. compounded half-yearly for 4 years. Find the value.
10. Find the effective annual rate of 18% p.a. compounded monthly.





CHAPTER 10

Probability

10.1 Revision: complement

10.2 Independent events

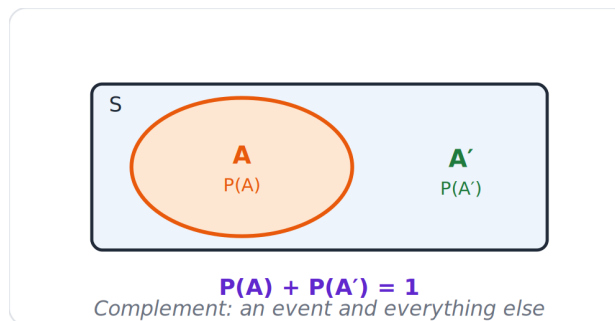
10.2 Dependent events

10.3 Venn diagrams

10.4 Tree diagrams

10.5 Contingency tables

10.1 Revision: complement



DEFINITIONS

Complement The complement of A is "not A": $P(A') = 1 - P(A)$.

Sample space The set of all possible outcomes.

KEY FORMULAS

- $P(A) + P(\text{not } A) = 1$
- So $P(\text{not } A) = 1 - P(A)$.
- Use the complement when "not happening" is easier to count.

WORKED EXAMPLES

Example 1: $P(\text{it rains}) = 0.3$. Find $P(\text{it does not rain})$.

► $P(\text{no rain}) = 1 - 0.3 = 0.7$.

Complement rule.



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**Applied example — Chance of a dry match**

The weather app gives a 30% chance of rain for Lerato's match. What is the chance it stays dry?

► $P(\text{dry}) = 1 - P(\text{rain})$

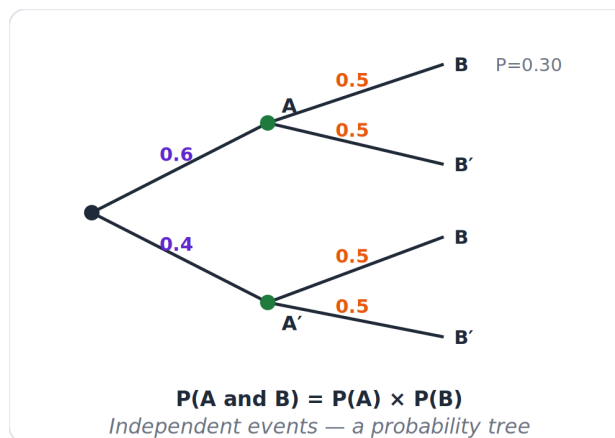
Complement rule.

► $= 1 - 0.30 = 0.70$ (70%)

Evaluate.

EXERCISES (EASY → DIFFICULT)

1. $P(A) = 0.3$. Find $P(\text{not } A)$.
2. $P(\text{rain}) = 0.25$. Find $P(\text{no rain})$.
3. A die is rolled. Find $P(\text{not a } 6)$.
4. $P(A) = 0.45$. Find $P(A')$.
5. A bag has 3 red and 7 blue. Find $P(\text{not red})$.
6. The weather app says 30% chance of rain for Lerato's match. What is the chance it stays dry?
7. A spinner lands on red with $P = 0.4$. Find $P(\text{not red})$.
8. In a class, $P(\text{a learner walks to school}) = 0.65$. Find $P(\text{does not walk})$.
9. $P(\text{passing a test}) = 0.8$. Out of 200 learners, how many are expected to fail?
10. A factory's machine is faulty 4% of the time. Find $P(\text{working})$, and expected good items in 500.

10.2 Independent events**DEFINITIONS****Mean**

The arithmetic average — the sum of all values divided by how many values there are.

When to use

Best for symmetric data with no extreme outliers.

KEY FORMULAS

- Events A and B are **independent** if $P(A \cap B) = P(A) \times P(B)$.
- Knowing one event happened does NOT change the probability of the other.
- Example: two coin tosses are independent.

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WORKED EXAMPLES

Example 1: A fair coin and a fair die are tossed. P(heads AND a 6)?

► $P(H) = 1/2$, $P(6) = 1/6$.

Independent events.

► $P(H \cap 6) = 1/2 \times 1/6 = 1/12$.

Multiply the probabilities.

Applied example — Passing both subjects

A learner passes Maths with $P = 0.7$ and English with $P = 0.6$, independently. Find $P(\text{passes both})$.

► $P(A \text{ and } B) = P(A) \cdot P(B)$

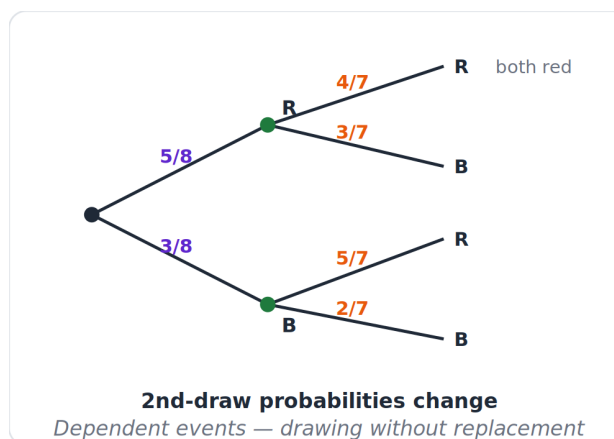
Independent events multiply.

► $= 0.7 \times 0.6 = 0.42$

Evaluate.

EXERCISES (EASY → DIFFICULT)

- For independent events, $P(A \text{ and } B) = ?$
- $P(A) = 0.5$, $P(B) = 0.4$, independent. Find $P(A \text{ and } B)$.
- Two coins are tossed. Find $P(\text{two heads})$.
- A die is rolled twice. Find $P(\text{two sixes})$.
- $P(A) = 0.6$, $P(B) = 0.5$, independent. Find $P(A \text{ and } B)$.
- A learner passes Maths with $P = 0.7$ and English with $P = 0.6$, independently. Find $P(\text{passes both})$.
- Two robots show red independently with $P = 0.4$ each. Find $P(\text{both red})$.
- A coin is tossed and a die rolled. Find $P(\text{heads and a 6})$.
- $P(A) = 0.7$, $P(A \text{ and } B) = 0.28$, A and B independent. Find $P(B)$.
- A machine produces a good part with $P = 0.95$. Find $P(\text{three good parts in a row})$.

10.2 Dependent events**DEFINITIONS****Median**

The MIDDLE value when the data is arranged in order. For an even count, the average of the two middle values.

Why use it

The median is NOT affected by outliers — better than mean for skewed data.

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KEY FORMULAS

- Events are **dependent** when one DOES affect the other.
- Use **conditional probability**: $P(A \cap B) = P(A) \times P(B | A)$.
- Classic example: drawing two cards from a deck WITHOUT replacing.

WORKED EXAMPLES

Example 1: A bag has 4 red and 6 blue balls. Two are drawn WITHOUT replacing. P(both red)?

► $P(\text{1st red}) = 4/10$. After that, 3 red of 9 remain.

Pool shrinks.

► $P(\text{both red}) = 4/10 \times 3/9 = 12/90 = 2/15$.

Multiply.

Applied example — Two reds without replacement

A bag has 6 red and 4 green sweets. Naledi eats two at random. Find $P(\text{both red})$.

► $P(\text{1st red}) = 6/10$

Six of ten are red.

► $P(\text{2nd red} | \text{1st red}) = 5/9$

One red gone, nine left.

► $P(\text{both}) = (6/10)(5/9) = 1/3$

Multiply (dependent).

EXERCISES (EASY → DIFFICULT)

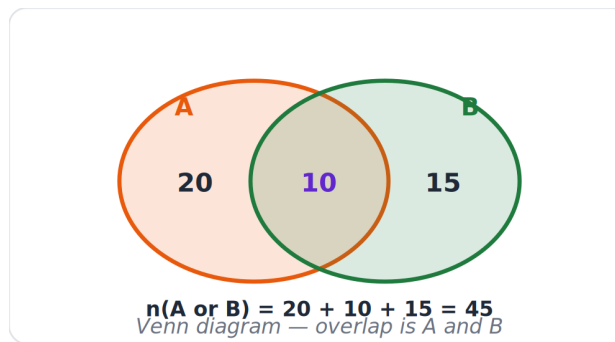
1. A bag has 2 red, 1 blue. Two drawn without replacement. Find $P(\text{both red})$.
2. From 5 cards (3 win), 2 are drawn without replacement. Find $P(\text{both winners})$.
3. Are draws without replacement independent or dependent?
4. A box has 4 good and 1 faulty bulb. Two are taken without replacement. Find $P(\text{both good})$.
5. From 3 red and 2 blue, two are drawn without replacement. Find $P(\text{first red, second blue})$.
6. A class has 12 girls and 8 boys. Two learners are chosen at random (no replacement). Find $P(\text{both girls})$.
7. A bag of sweets has 6 red and 4 green. Naledi eats two at random. Find $P(\text{both red})$.
8. From a deck of 52, two cards are drawn without replacement. Find $P(\text{both aces})$.
9. A drawer has 5 black and 3 white socks. Two are taken without looking. Find $P(\text{a matching pair})$.
10. From 10 raffle tickets (2 winning), Thabo takes 2 without replacement. Find $P(\text{at least one winner})$.



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10.3 Venn diagrams



DEFINITIONS

Frequency	How many times a value or class appears in a data set.
Class interval	A range of values, e.g. $0 \leq x < 10$. Used when data has many distinct values.
Frequency table	A table showing each class and its frequency, used to organise large data sets.

KEY FORMULAS

- Two circles represent two events A and B.
- Overlap = $A \cap B$ (both happen).
- Sum of all regions = total outcomes in the sample space.
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

WORKED EXAMPLES

Example 1: In a class of 30, 18 take Maths, 12 take Science, 8 take both. How many take at least one?

► $|M \cup S| = 18 + 12 - 8 = 22$.

Inclusion-exclusion.

► Eight take neither ($30 - 22$).

Outside both circles.

Applied example — Subjects in a class

In a class of 40, 22 do Maths, 18 do Science, 8 do both. How many do neither?

► $n(M \cup S) = 22 + 18 - 8 = 32$

Inclusion-exclusion.

► Neither = $40 - 32 = 8$

Subtract from the total.

EXERCISES (EASY → DIFFICULT)

1. In a Venn diagram, what does the overlap of A and B represent?
2. $n(A) = 20$, $n(B) = 15$, $n(A \cap B) = 5$. Find $n(A \cup B)$.
3. Out of 50, 30 like tea, 25 like coffee, 10 like both. How many like at least one?
4. Using the above, how many like neither?
5. $n(A \text{ only})$ when $n(A) = 18$ and $n(A \cap B) = 7$.
6. In a class of 40, 22 do Maths, 18 do Science, 8 do both. How many do only Maths?

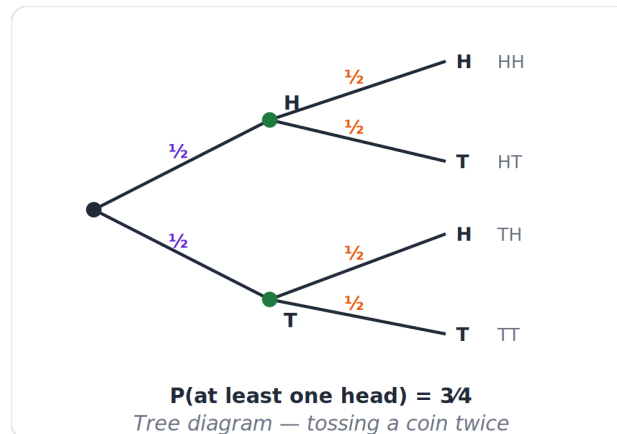


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7. From the same class, how many do neither subject?
8. Of 100 shoppers, 60 buy bread, 45 buy milk, 25 buy both. Find $P(\text{buys bread or milk})$.
9. In a survey of 80, 50 own a phone, 30 own a tablet, 20 own both. How many own exactly one device?
10. A and B are mutually exclusive with $n(A) = 12$, $n(B) = 9$ out of 30. Find $P(A \text{ or } B)$.

10.4 Tree diagrams



DEFINITIONS

- Tree diagram** A branching diagram of outcomes; multiply probabilities along branches.
- Branch** One path representing a single outcome.

KEY FORMULAS

- Each branch shows an outcome and its probability.
- Multiply along a path to get a joint probability.
- Add probabilities of paths leading to the same final outcome.
- All probabilities at each branching point sum to 1.

WORKED EXAMPLES

Example 1: A coin is tossed twice. $P(\text{exactly one head})$?

- Paths: HT and TH. Each has probability $\frac{1}{4}$.

Multiply $\frac{1}{2} \times \frac{1}{2}$ along each path.

- $P(\text{exactly one head}) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

Add the path probabilities.

Applied example — A two-stage draw

A bag has 4 red and 1 green. Two are drawn without replacement. Find $P(\text{one of each colour})$.

- $P(R \text{ then } G) = \left(\frac{4}{5}\right)\left(\frac{1}{4}\right) = \frac{4}{20}$

First branch of the tree.

- $P(G \text{ then } R) = \left(\frac{1}{5}\right)\left(\frac{4}{4}\right) = \frac{4}{20}$

Second branch.

- Total = $\frac{8}{20} = \frac{2}{5}$

Add the branches.



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**EXERCISES (EASY → DIFFICULT)**

1. A coin is tossed twice. How many branches end the tree?
2. A coin is tossed 3 times. How many outcomes?
3. On a tree, probabilities along a branch are combined by...
4. A coin is tossed twice. Find $P(\text{two heads})$ using a tree.
5. A spinner (red, blue) is spun twice, $P(\text{red}) = 0.6$. Find $P(\text{red then blue})$.
6. A bag has 3 red, 2 blue. Two drawn without replacement; use a tree to find $P(\text{both red})$.
7. A learner guesses a 3-question true/false quiz. Use a tree to find $P(\text{all three correct})$.
8. Two robots: each is red with $P = 0.4$. Use a tree to find $P(\text{exactly one red})$.
9. A test is 90% accurate. Two independent tests are run. Find $P(\text{both correct})$.
10. A bag has 4 red, 1 green. Two are drawn without replacement. Find $P(\text{one of each colour})$.

10.5 Contingency tables

	Sci ✓	Sci ✗	Total
Math ✓	25	35	60
Math ✗	15	25	40
Total	40	60	100

*Contingency (two-way) table with totals***DEFINITIONS****Five-number summary**

The five numbers that fully describe a data set's shape: minimum, Q_1 , median (Q_2), Q_3 , maximum.

Box-and-whisker plot

A visual diagram of the five-number summary — the box runs from Q_1 to Q_3 with a line at the median.

KEY FORMULAS

- Rows represent one variable, columns another.
- Cell shows the count for that combination.
- Row totals + column totals = grand total.
- $P(A | B) = (\text{cell value}) / (\text{column total of } B)$.

WORKED EXAMPLES**Example 1: In a survey of 100 people: 40 like coffee (15 of whom also like tea), 60 like tea. $P(\text{coffee} | \text{tea})$?**

- Build the table: 15 like both, so 25 like only coffee, 45 like only tea, 15 like neither.

Fill in totals.

- $P(\text{coffee} | \text{tea}) = 15 / 60 = 1/4$.

Cell ÷ column total.

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**Applied example — A contingency table**

Of 200 people, 120 are adults; 90 adults and 20 children own a phone. Find $P(\text{owns a phone})$.

► Phone owners = $90 + 20 = 110$

Add across the table.

► $P = 110/200 = 0.55$

Divide by the grand total.

EXERCISES (EASY → DIFFICULT)

1. A contingency table shows counts for two variables. What does the grand total equal?
2. 80 people: 50 male, 30 female. $P(\text{female})$?
3. A table shows 40 pass, 10 fail out of 50. Find $P(\text{pass})$.
4. Of 100 people, 60 like soccer; 35 of those are male. Find $P(\text{male and likes soccer})$.
5. For independence, $P(A \text{ and } B)$ should equal...
6. In a survey of 200, 120 are adults and 80 children; 90 adults own a phone, 20 children do. Find $P(\text{owns a phone})$.
7. Using that table, find $P(\text{adult and owns a phone})$.
8. Test if "being an adult" and "owning a phone" are independent: $P(\text{adult}) = 0.6$, $P(\text{phone}) = 0.55$, $P(\text{both}) = 0.45$. Independent?
9. A clinic table: 70 of 100 vaccinated did not get flu; 10 of 30 unvaccinated did not. Find $P(\text{did not get flu})$.
10. In a class of 40, 24 do sport and of those 18 are boys; there are 25 boys. Find $P(\text{a boy does sport})$.

End of Chapter 10 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. $P(A) = 0.3$. Find $P(\text{not } A)$.
2. $P(A) = 0.5$, $P(B) = 0.4$, independent. Find $P(A \text{ and } B)$.
3. Are draws without replacement independent or dependent?
4. Using the above, how many like neither?
5. A spinner (red, blue) is spun twice, $P(\text{red}) = 0.6$. Find $P(\text{red then blue})$.
6. In a survey of 200, 120 are adults and 80 children; 90 adults own a phone, 20 children do. Find $P(\text{owns a phone})$.
7. A spinner lands on red with $P = 0.4$. Find $P(\text{not red})$.
8. A coin is tossed and a die rolled. Find $P(\text{heads and a } 6)$.
9. A drawer has 5 black and 3 white socks. Two are taken without looking. Find $P(\text{a matching pair})$.
10. A and B are mutually exclusive with $n(A) = 12$, $n(B) = 9$ out of 30. Find $P(A \text{ or } B)$.



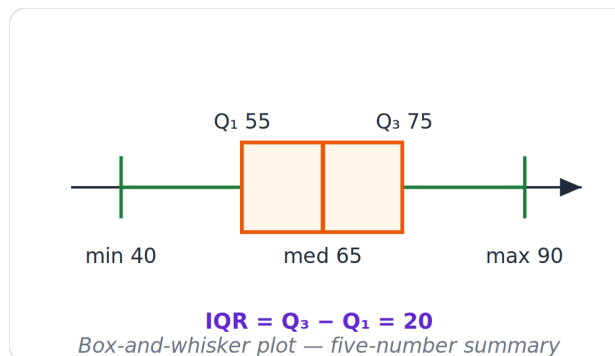


CHAPTER 11

Statistics

- 11.1** Revision: 5-number summary
- 11.2** Histograms
- 11.3** Ogives (cumulative-frequency curves)
- 11.4** Variance
- 11.4** Standard deviation
- 11.5** Symmetric vs skewed data
- 11.6** Identification of outliers

11.1 Revision: 5-number summary



DEFINITIONS

- | | |
|----------------------------|--|
| Five-number summary | Minimum, Q_1 , median, Q_3 , maximum. |
| Quartiles | Values splitting ordered data into four equal parts. |

KEY FORMULAS

- **5-number summary:** min, Q_1 , median, Q_3 , max
- **IQR** (interquartile range) = $Q_3 - Q_1$
- Box-and-whisker shows all 5 numbers visually.



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WORKED EXAMPLES

Example 1: Data: 3, 5, 7, 8, 12, 15, 19. Find the 5-number summary.

- Sorted: 3, 5, 7, 8, 12, 15, 19. Median (middle of 7) = 8.

Already sorted.

- Lower half: 3, 5, 7 → $Q1 = 5$. Upper half: 12, 15, 19 → $Q3 = 15$.

Quartiles.

- 5-number summary: 3, 5, 8, 15, 19. IQR = 10.

Done.

Applied example — A box plot

Test marks: 40, 55, 60, 65, 70, 75, 90. Give the five-number summary and the IQR.

- min 40, $Q1$ 55, median 65, $Q3$ 75, max 90

Order and split the data.

- $IQR = 75 - 55 = 20$

$Q3 - Q1$.

EXERCISES (EASY → DIFFICULT)

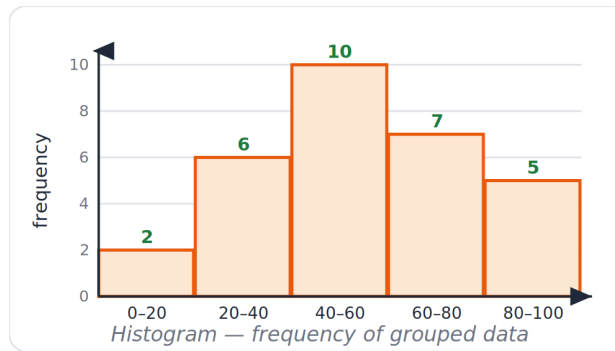
1. Find the median of 3, 7, 8, 12, 15.
2. Find the minimum and maximum of 4, 9, 2, 15, 7.
3. Find the range of 5, 8, 12, 20.
4. Find $Q1$ of 2, 4, 6, 8, 10, 12, 14, 16.
5. Find the interquartile range (IQR) when $Q1 = 5$ and $Q3 = 13$.
6. Test marks: 40, 55, 60, 65, 70, 75, 90. Give the five-number summary.
7. For the marks above, find the IQR.
8. Daily sales (R): 200, 350, 400, 450, 600, 700. Find the median.
9. Find the five-number summary of 10, 12, 14, 16, 18, 20, 22, 24.
10. A box-and-whisker plot shows min 12, $Q1$ 20, median 28, $Q3$ 35, max 50. Find the IQR and range.

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11.2 Histograms



DEFINITIONS

- Histogram** A bar graph of grouped (continuous) data with no gaps.
- Class interval** A range of values grouped into one bar.

KEY FORMULAS

- X-axis: class intervals (groups of values).
- Y-axis: frequency (count) in each class.
- Bars touch — there are no gaps in continuous data.
- The shape of the histogram tells you the distribution.

WORKED EXAMPLES

Example 1: Heights (cm): 150, 152, 155, 158, 161, 162, 163, 167, 170, 178. Use classes of width 10.

- Classes: 150-160 (5), 160-170 (4), 170-180 (1).

Count each.

- Draw three touching bars with heights 5, 4, 1.

That's the histogram.

Applied example — Reading a histogram

Marks group as [0-20):2, [20-40):6, [40-60):10, [60-80):7, [80-100):5. How many scored 40 or more?

- Add the last three bars: $10 + 7 + 5$

Frequencies for 40 and above.

- = 22 learners

Sum.

EXERCISES (EASY → DIFFICULT)

1. In a histogram, what does the height of each bar show?
2. Bars: [0-10): 4, [10-20): 7, [20-30): 9. How many data values in total?
3. Why are histogram bars drawn touching (no gaps)?
4. Which interval is the modal class if [20-30) has the highest bar?
5. Frequencies 5, 8, 12, 6, 3. Which class is the mode?
6. Heights (cm) of plants: [0-5): 3, [5-10): 8, [10-15): 5. How many plants are under 10 cm?
7. Using the plant data, what is the modal class?



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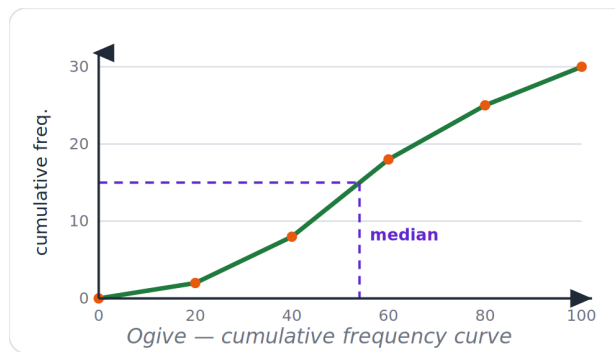
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8. Marks grouped [0–20): 2, [20–40): 6, [40–60): 10, [60–80): 7, [80–100): 5. How many learners?
9. From the marks above, how many scored 40 or more?
10. Estimate the mode interval and say what shape the marks suggest if frequencies rise then fall.

11.3 Ogives

Ogives (cumulative-frequency curves)



DEFINITIONS

Ogive	A cumulative frequency curve.
Cumulative frequency	A running total of frequencies.

KEY FORMULAS

- X-axis: data value (class boundaries).
- Y-axis: CUMULATIVE frequency (running total).
- Plot points at the UPPER boundary of each class.
- Read off medians and quartiles from the curve.

WORKED EXAMPLES

Example 1: Ogive shows total 40, and the cumulative frequency hits 20 at $x = 15$. What's the median?

- Median is at 50% — that's 20 out of 40.
Halfway up.
- Median ≈ 15 .
Read across from $y = 20$ to the curve, then down.

Applied example — Reading an ogive

An ogive for 80 runners shows 20 finished under 40 minutes. What fraction is that?

- Fraction = $20/80$
Cumulative count over total.
- $= 1/4$
Simplify.



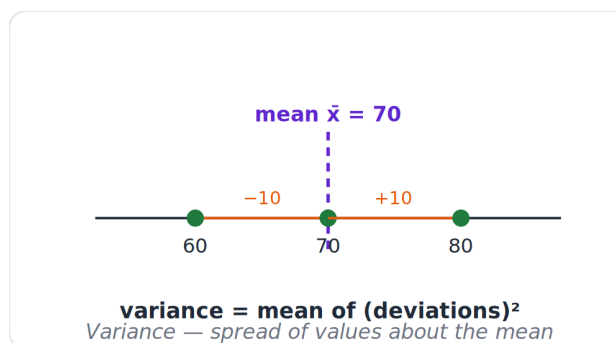
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**EXERCISES (EASY → DIFFICULT)**

1. What does an ogive plot?
2. Cumulative frequencies are 4, 11, 20, 25. What is the total number of data?
3. On an ogive of 40 values, the median is read at which cumulative frequency?
4. Frequencies 5, 10, 8, 7. Write the cumulative frequencies.
5. From an ogive, the value at cumulative frequency 25% of the total gives...
6. 60 learners' marks: cumulative frequencies reach 30 at 55%. Estimate the median mark.
7. An ogive for 80 runners shows 20 finished under 40 min. What fraction is that?
8. Cumulative: [0-10):3, [0-20):10, [0-30):24, [0-40):30. How many lie in [20-30)?
9. From an ogive of 100 values, Q1 reads at the 25th value and Q3 at the 75th. What is the IQR if these are 30 and 62?
10. On an ogive of 50 plants, 40 are under 12 cm. What percentage is 12 cm or taller?

11.4 Variance

**DEFINITIONS**

Variance	The average of the squared deviations from the mean.
Deviation	How far a value is from the mean.

KEY FORMULAS

- **Variance (population):** $\sigma^2 = \Sigma(x_i - \bar{x})^2 / n$
- It measures average **squared** deviation from the mean.
- Always non-negative. Same units squared as the original data.

WORKED EXAMPLES**Example 1: Find the variance of 4, 6, 8, 10.**

► Mean = 7. Deviations: -3, -1, 1, 3.

From the mean.

► Squares: 9, 1, 1, 9. Sum = 20. Variance = $20/4 = 5$.

$\sigma^2 = 5$.

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**Applied example — Variance of marks**

A learner's marks are 60, 70, 80. Find the variance.

► Mean = 70

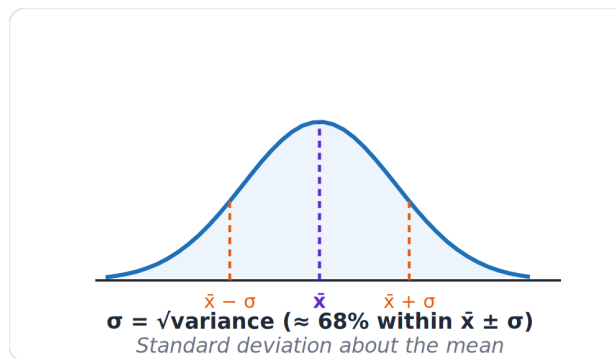
Add and divide by 3.

► Variance = $((-10)^2 + 0^2 + 10^2)/3 = 200/3 \approx 66.67$

Mean of squared deviations.

EXERCISES (EASY → DIFFICULT)

1. Define variance in words.
2. Find the mean of 2, 4, 6.
3. Find the variance of 2, 4, 6.
4. Find the variance of 1, 3, 5, 7.
5. Find the variance of 5, 5, 5, 5.
6. A learner's marks are 60, 70, 80. Find the variance.
7. Rainfall (mm): 2, 4, 6, 8. Find the variance.
8. Which data set has greater variance: {4,5,6} or {1,5,9}?
9. Find the variance of 10, 12, 14, 16, 18.
10. If every value in a data set increases by 5, what happens to the variance?

11.4 Standard deviation**DEFINITIONS****Standard deviation**

The square root of the variance — spread in original units.

Spread

How widely the data is dispersed.

KEY FORMULAS

- **Standard deviation:** $\sigma = \sqrt{\text{variance}}$
- Same units as the original data.
- Roughly: average distance of points from the mean.

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WORKED EXAMPLES

Example 1: Variance is 9. Find the standard deviation.

► $\sigma = \sqrt{9} = 3.$

Same units as the data.

Applied example — Comparing consistency

Two classes share a mean. Class A has $\sigma = 5$, Class B has $\sigma = 12$. Which is more consistent?

► Smaller $\sigma \Rightarrow$ less spread

Standard deviation measures spread.

► Class A is more consistent

$\sigma = 5 < 12.$

EXERCISES (EASY → DIFFICULT)

- How is the standard deviation related to the variance?
- Variance is 16. Find the standard deviation.
- Variance is 25. Find the standard deviation.
- Find the standard deviation of 2, 4, 6.
- A data set has $\sigma = 0$. What does that tell you?
- Test marks 60, 70, 80 have variance $200/3$. Find the standard deviation (2 dp).
- Two classes have the same mean; class A $\sigma = 5$, class B $\sigma = 12$. Which is more consistent?
- Find the standard deviation of 1, 3, 5, 7 (2 dp).
- A factory fills bottles with mean 500 mL, $\sigma = 4$ mL. About what range holds ~68% of bottles ($\text{mean} \pm 1\sigma$)?
- If every value is multiplied by 3, what happens to the standard deviation?

11.5 Symmetric vs skewed data**DEFINITIONS****Symmetric data**

Data balanced about the mean ($\text{mean} \approx \text{median}$).

Skewed data

Data with a longer tail on one side.

KEY FORMULAS**DIGITAL TEXTBOOK**

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- **Symmetric:** mean $\approx$ median.
- **Skewed right (positive):** long right tail, mean $>$ median.
- **Skewed left (negative):** long left tail, mean $<$ median.
- Comparing mean and median is the quickest test.

WORKED EXAMPLES

Example 1: Test scores: most students in 70s and 80s, a few in 30s. Skewed which way?

- Low outliers create a long LEFT tail.

Skew direction = direction of the long tail.

- So this is **skewed left** (negative skew).

Mean will be slightly below the median.

Applied example — Skewness of incomes

Incomes in a town have a few very high earners. Describe the skew and order mean/median.

- A long right tail $\Rightarrow$ positive (right) skew

Few large values pull the tail right.

- mode $<$ median $<$ mean

The mean is dragged upward.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

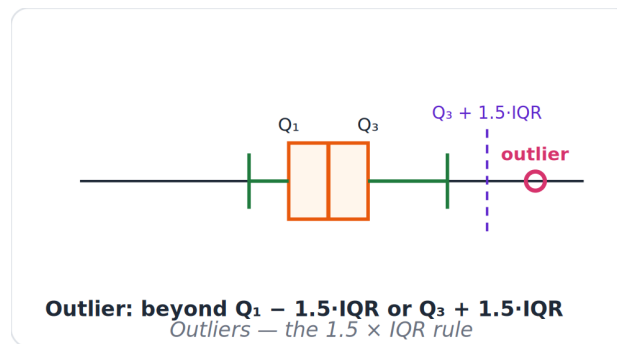
1. If mean = median = mode, the distribution is...
2. If the mean is greater than the median, the data is skewed which way?
3. If the mean is less than the median, the data is skewed which way?
4. A long tail to the right indicates which skew?
5. Mean 50, median 50, mode 50. Describe the shape.
6. Incomes have a few very high earners. What skew is expected?
7. Mean 62, median 70 for a test. Describe the skew.
8. Exam marks bunch near the top with a tail of low marks. What skew?
9. In a right-skewed data set, order mean, median and mode from smallest.
10. A box plot has the median line close to Q1 with a long upper whisker. What skew?



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11.6 Identification of outliers



DEFINITIONS

Outlier A value far from the rest, beyond $Q_1 - 1.5 \cdot IQR$ or $Q_3 + 1.5 \cdot IQR$.

IQR Interquartile range = $Q_3 - Q_1$.

KEY FORMULAS

- **$1.5 \times IQR$ rule:** any value below $Q_1 - 1.5 \times IQR$ or above $Q_3 + 1.5 \times IQR$ is an outlier.
- Compute Q_1 , Q_3 , IQR first.
- Then check both fences (lower and upper).

WORKED EXAMPLES

Example 1: $Q_1 = 10$, $Q_3 = 18$ ($IQR = 8$). Is 32 an outlier?

► Upper fence = $18 + 1.5 \times 8 = 30$.

Standard $1.5 \times IQR$.

► $32 > 30$, so YES, it is an outlier.

Beyond the upper fence.

Applied example — Spotting an outlier

Marks have $Q_1 = 50$ and $Q_3 = 70$. Is a mark of 110 an outlier?

► $IQR = 20$; upper bound = $Q_3 + 1.5 \cdot IQR = 70 + 30 = 100$

The $1.5 \times IQR$ rule.

► $110 > 100 \rightarrow$ yes, it is an outlier

Compare.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. State the $1.5 \times IQR$ rule for an outlier on the high side.
2. $Q_1 = 10$, $Q_3 = 20$. Find the IQR .
3. With $Q_1 = 10$, $Q_3 = 20$, find the upper outlier boundary.
4. With $Q_1 = 10$, $Q_3 = 20$, find the lower outlier boundary.
5. Is 40 an outlier if the upper boundary is 35?
6. Marks have $Q_1 = 50$, $Q_3 = 70$. Is a mark of 95 an outlier?
7. Daily sales have $Q_1 = R300$, $Q_3 = R500$. Find the outlier boundaries.
8. Is a sales day of R900 an outlier (boundaries R0 and R800)?
9. A data set has $Q_1 = 12$, $Q_3 = 28$. Identify which of 0, 5, 55 are outliers.
10. Why use the $1.5 \times IQR$ rule rather than just the largest value?



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End of Chapter 11 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Find the median of 3, 7, 8, 12, 15.
2. Bars: [0–10): 4, [10–20): 7, [20–30): 9. How many data values in total?
3. On an ogive of 40 values, the median is read at which cumulative frequency?
4. Find the variance of 1, 3, 5, 7.
5. A data set has $\sigma = 0$. What does that tell you?
6. Incomes have a few very high earners. What skew is expected?
7. Daily sales have $Q1 = R300$, $Q3 = R500$. Find the outlier boundaries.
8. Daily sales (R): 200, 350, 400, 450, 600, 700. Find the median.
9. From the marks above, how many scored 40 or more?
10. On an ogive of 50 plants, 40 are under 12 cm. What percentage is 12 cm or taller?





CHAPTER 12

Linear programming

12.1 Linear programming: identify constraints

12.1 Optimisation: corner points

12.1 Evaluate the objective at corners

12.1 Linear programming: identify constraints

DEFINITIONS

Parallelogram A quadrilateral with both pairs of opposite sides parallel.

Diagonal A line joining two non-adjacent vertices.

Bisect To cut into two EQUAL halves.

KEY FORMULAS

- Define the decision variables (usually x and y representing quantities).
- Each restriction becomes a linear inequality.
- Don't forget the non-negativity constraints $x \geq 0$, $y \geq 0$.
- Graph all inequalities — feasible region is where they all overlap.

WORKED EXAMPLES

Example 1: "Make at most 40 chairs and tables combined, with at most 25 tables."
Write the constraints.

► Let c = chairs, t = tables.

Define variables.

► $c + t \leq 40$, $t \leq 25$, $c \geq 0$, $t \geq 0$.

Four constraints.

Applied example — Baker's constraints

A baker makes x loaves and y cakes, at most 30 items a day; a loaf needs 2 cups of flour and a cake 3, with 60 cups available. Write the constraints.

► $x + y \leq 30$

Total items limit.

► $2x + 3y \leq 60$

Flour limit.

► $x \geq 0$, $y \geq 0$

Cannot make a negative number.



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**EXERCISES (EASY → DIFFICULT)**

1. Write an inequality for "x is at most 8".
2. Write an inequality for "y is at least 3".
3. Write an inequality for "x and y cannot be negative".
4. Write an inequality for "the total of x and y is no more than 10".
5. Write an inequality for "at least twice as many x as y".
6. A baker makes x loaves and y cakes. Each day she can make at most 30 items in total. Write the constraint.
7. A loaf needs 2 cups of flour and a cake needs 3; only 60 cups are available. Write the flour constraint.
8. Thabo sells x caps and y shirts; he must sell at least 5 shirts. Write the constraint.
9. A factory makes x tables and y chairs, needing at least 3 chairs per table. Write the constraint.
10. A minibus carries at most 14 passengers and at least 6 to run. Write both constraints.

12.1 Optimisation: corner points

DEFINITIONS**Rhombus**

A parallelogram with all FOUR sides equal in length.

Perpendicular bisectors

Two lines that cross at 90° AND each cuts the other in half.

KEY FORMULAS

- The **feasible region** is the intersection of all the constraint half-planes.
- It's always a convex polygon (or unbounded).
- The **maximum/minimum** of a linear objective always lies at a CORNER.
- So only the corner points need to be checked.

WORKED EXAMPLES

Example 1: Corners of feasible region: (0,0), (4,0), (3,2), (0,3). Maximise $z = 2x + 3y$.

- Evaluate z at each corner: 0, 8, 12, 9.

Try every corner.

- Max $z = 12$ at (3, 2).

Pick the biggest.

Applied example — Finding a corner

A feasible region includes the lines $x + y = 10$ and $2x + y = 14$. Find their corner point.

- Subtract: $(2x + y) - (x + y) = 14 - 10 \rightarrow x = 4$

Eliminate y.

- $y = 10 - 4 = 6 \rightarrow (4, 6)$

Back-substitute.

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**EXERCISES (EASY → DIFFICULT)**

1. What is a "corner point" (vertex) of a feasible region?
2. Find where $x + y = 6$ meets $x = 2$.
3. Find where $y = x$ meets $x + y = 8$.
4. Find where $x = 0$ meets $2x + y = 10$.
5. Find where $y = 0$ meets $x + 2y = 8$.
6. A feasible region is bounded by $x \geq 0$, $y \geq 0$, $x + y \leq 6$. List its corner points.
7. Find the corner where $x + y = 10$ meets $2x + y = 14$.
8. Find where $3x + 2y = 12$ meets $x = 2$.
9. A region has $x \geq 0$, $y \geq 0$, $x \leq 5$, $y \leq 4$. List all four corners.
10. Find the intersection of $x + y = 7$ and $y = 2x + 1$.

12.1 Optimisation: evaluate objective at corners*Evaluate the objective at corners***DEFINITIONS****Objective function**

The quantity (e.g. profit) being maximised or minimised.

Corner-point principle

The optimum lies at a vertex of the feasible region.

KEY FORMULAS

- Substitute each corner (x, y) into the objective $z = ax + by$.
- Tabulate the results.
- Pick the largest (maximise) or smallest (minimise).

WORKED EXAMPLES**Example 1: Corners (0,0), (5,0), (4,3). Minimise $z = x + 4y$.**► z values: 0, 5, 16.*Substitute each corner.*

► Minimum is 0 at (0, 0).

*Smallest value.***Applied example — Maximum profit**A baker's profit is $P = 8x + 5y$. The corners are (0,30), (10,20), (20,0). Find the maximum profit.► $P(0,30)=150$, $P(10,20)=180$, $P(20,0)=160$ *Evaluate P at each corner.*

► Maximum = R180 at (10, 20)

*Pick the largest.***EXERCISES (EASY → DIFFICULT)**

1. Evaluate $P = 2x + 3y$ at (4, 1).
2. Evaluate $P = x + y$ at (5, 2).
3. Evaluate $C = 3x + 2y$ at (0, 6).
4. Evaluate $P = 5x + 4y$ at (2, 3).

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5. At which corner is $P = x + y$ largest: (0,6), (6,0) or (3,3)?
6. A profit $P = 40x + 30y$ is checked at (0,30), (20,10), (30,0). Find the maximum.
7. Corner points are (0,0), (4,0), (0,5), (2,3). Maximise $P = 3x + 2y$.
8. Minimise $C = 2x + 5y$ over corners (1,4), (3,2), (6,1).
9. A baker's profit is $P = 8x + 5y$. Corners are (0,30), (10,20), (20,0). Find the maximum profit.
10. Why does the optimum of a linear objective always occur at a corner point?

End of Chapter 12 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Write an inequality for "x is at most 8".
2. Find where $x + y = 6$ meets $x = 2$.
3. Evaluate $C = 3x + 2y$ at (0, 6).
4. Write an inequality for "the total of x and y is no more than 10".
5. Find where $y = 0$ meets $x + 2y = 8$.
6. A profit $P = 40x + 30y$ is checked at (0,30), (20,10), (30,0). Find the maximum.
7. A loaf needs 2 cups of flour and a cake needs 3; only 60 cups are available. Write the flour constraint.
8. Find where $3x + 2y = 12$ meets $x = 2$.
9. A baker's profit is $P = 8x + 5y$. Corners are (0,30), (10,20), (20,0). Find the maximum profit.
10. A minibus carries at most 14 passengers and at least 6 to run. Write both constraints.

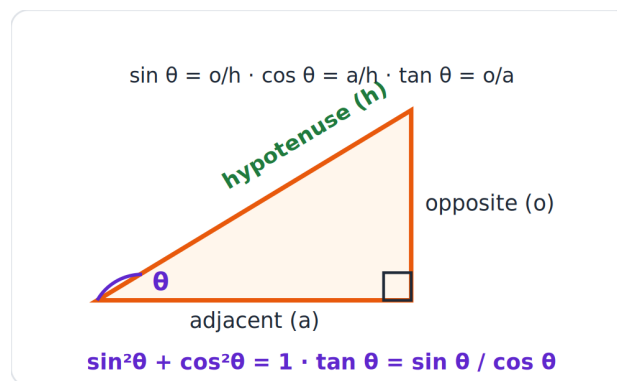


CHAPTER 6

Trigonometry

- 11.1** Trigonometric identities
- 11.2** Reduction formula
- 11.3** Trigonometric equations
- 11.4** Area, sine, and cosine rules

11.1 Trigonometric identities



DEFINITIONS

Trigonometric identity An equation true for all angles, e.g. $\sin^2 \theta + \cos^2 \theta = 1$.

Quotient identity $\tan \theta = \sin \theta / \cos \theta$.

KEY FORMULAS

- **Pythagorean identity:** $\sin^2 \theta + \cos^2 \theta = 1$
- **Quotient identity:** $\tan \theta = \sin \theta \div \cos \theta$
- **Variants:** $\sin^2 \theta = 1 - \cos^2 \theta$; $\cos^2 \theta = 1 - \sin^2 \theta$
- These hold for every angle θ where the expressions are defined.



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Applied example — Simplifying an identity

Simplify $(1 - \cos^2\theta)/\sin\theta$.

► $1 - \cos^2\theta = \sin^2\theta$

Pythagorean identity.

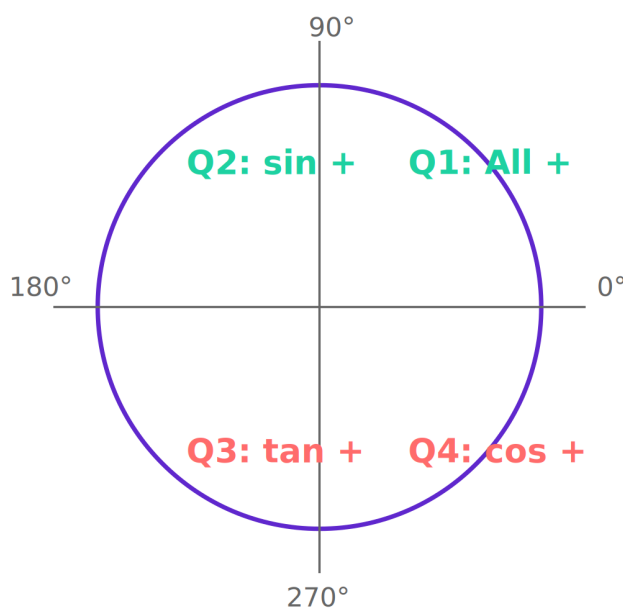
► $\sin^2\theta/\sin\theta = \sin\theta$

Cancel.

EXERCISES (EASY → DIFFICULT)

1. Simplify $\sin^2\theta + \cos^2\theta$.
2. Write $\tan\theta$ in terms of $\sin\theta$ and $\cos\theta$.
3. Simplify $1 - \cos^2\theta$.
4. Simplify $(\sin\theta \cdot \cos\theta)/\sin\theta$.
5. Simplify $\sin\theta/\tan\theta$.
6. Simplify $(1 - \cos^2\theta)/\sin\theta$.
7. Prove $\tan\theta \cdot \cos\theta = \sin\theta$.
8. Simplify $\cos^2\theta - 1$.
9. Simplify $(\sin^2\theta)/(1 - \cos\theta)$.
10. If $\sin\theta = 3/5$ and θ is acute, find $\cos\theta$ and $\tan\theta$.

11.2 Reduction formula



DEFINITIONS

Reduction formula

A rule that rewrites a trig ratio of a large/negative angle as a ratio of an acute reference angle.

Reference angle

The acute angle to the x-axis used to evaluate the ratio.



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KEY FORMULAS

- **Q2 (90°-180°):** $\sin(180^\circ - \theta) = \sin \theta$, $\cos(180^\circ - \theta) = -\cos \theta$, $\tan(180^\circ - \theta) = -\tan \theta$
- **Q3 (180°-270°):** $\sin(180^\circ + \theta) = -\sin \theta$, $\cos(180^\circ + \theta) = -\cos \theta$, $\tan(180^\circ + \theta) = \tan \theta$
- **Q4 (270°-360°):** $\sin(360^\circ - \theta) = -\sin \theta$, $\cos(360^\circ - \theta) = \cos \theta$, $\tan(360^\circ - \theta) = -\tan \theta$
- **Negative angles:** $\sin(-\theta) = -\sin \theta$, $\cos(-\theta) = \cos \theta$, $\tan(-\theta) = -\tan \theta$
- **Co-functions (90°):** $\sin(90^\circ - \theta) = \cos \theta$, $\cos(90^\circ - \theta) = \sin \theta$

Applied example — Using a reduction formula

Evaluate $\cos 150^\circ$ exactly.

► $\cos 150^\circ = \cos(180^\circ - 30^\circ) = -\cos 30^\circ$

Reduction in the second quadrant.

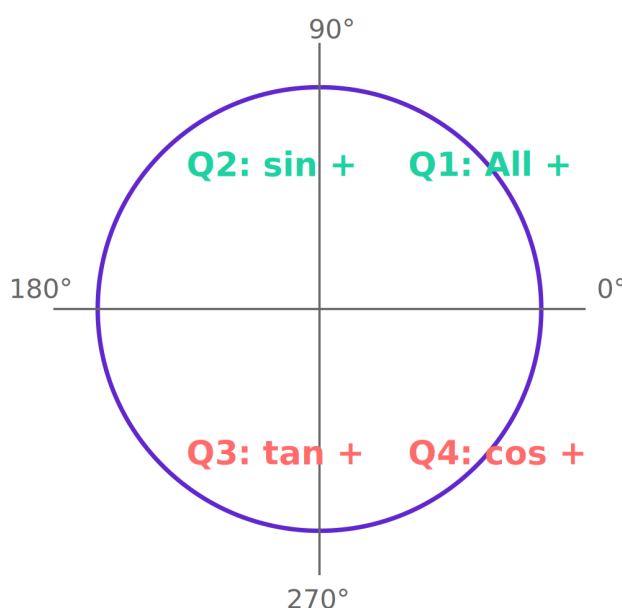
► $= -\sqrt{3}/2$

Exact value.

EXERCISES (EASY → DIFFICULT)

1. Simplify $\sin(180^\circ - \theta)$.
2. Simplify $\cos(180^\circ - \theta)$.
3. Simplify $\sin(360^\circ - \theta)$.
4. Simplify $\cos(-\theta)$.
5. Simplify $\tan(180^\circ + \theta)$.
6. Simplify $\sin(90^\circ - \theta)$.
7. Evaluate $\cos 150^\circ$ using a reduction formula.
8. Evaluate $\sin 210^\circ$.
9. Simplify $(\sin(180^\circ - \theta) \cdot \cos(360^\circ - \theta)) / \sin \theta$.
10. Evaluate $\cos 300^\circ + \sin 120^\circ$.

11.3 Trigonometric equations



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DEFINITIONS

Trigonometric equation	An equation solved for the angle, giving a general solution.
General solution	All angles satisfying the equation (adding multiples of the period).

KEY FORMULAS

- **$\sin \theta = k$:** $\theta = \sin^{-1}(k) + 360^\circ n$ OR $\theta = 180^\circ - \sin^{-1}(k) + 360^\circ n$
- **$\cos \theta = k$:** $\theta = \pm \cos^{-1}(k) + 360^\circ n$
- **$\tan \theta = k$:** $\theta = \tan^{-1}(k) + 180^\circ n$
- n is any integer. Restrict to the requested interval (e.g. $0^\circ \leq \theta < 360^\circ$).

Applied example — Solving a trig equation

Solve $2\sin\theta - 1 = 0$ for $0^\circ \leq \theta \leq 360^\circ$.

► $\sin\theta = \frac{1}{2}$

Isolate $\sin\theta$.

► $\theta = 30^\circ$ or 150°

Sine is positive in quadrants I and II.

EXERCISES (EASY → DIFFICULT)

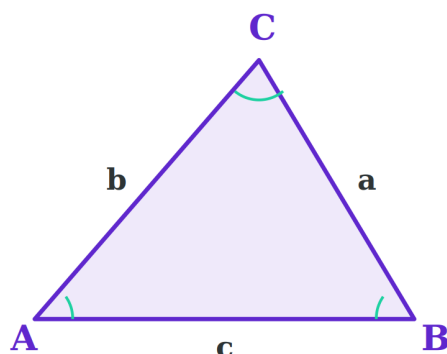
1. Solve $\sin\theta = \frac{1}{2}$ for $0^\circ \leq \theta \leq 360^\circ$.
2. Solve $\cos\theta = 0$ for $0^\circ \leq \theta \leq 360^\circ$.
3. Solve $\tan\theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$.
4. Solve $2\cos\theta = \sqrt{3}$ for $0^\circ \leq \theta \leq 360^\circ$.
5. Solve $\sin\theta = -\frac{1}{2}$ for $0^\circ \leq \theta \leq 360^\circ$.
6. Solve $2\sin\theta - 1 = 0$ for $0^\circ \leq \theta \leq 360^\circ$.
7. Solve $\tan\theta = \sqrt{3}$ for $0^\circ \leq \theta \leq 360^\circ$.
8. Solve $\cos\theta = -\frac{1}{2}$ for $0^\circ \leq \theta \leq 360^\circ$.
9. Solve $2\sin^2\theta = \sin\theta$ for $0^\circ \leq \theta \leq 360^\circ$.
10. Give the general solution of $\sin\theta = \frac{1}{2}$.



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11.4 Area, sine, and cosine rules



DEFINITIONS

Sine rule	$a/\sin A = b/\sin B = c/\sin C$ — relates sides to opposite angles.
Cosine rule	$a^2 = b^2 + c^2 - 2bc \cdot \cos A$ — used for SAS or SSS triangles.
Area rule	$\text{Area} = \frac{1}{2} \cdot ab \cdot \sin C$ — area from two sides and the included angle.

KEY FORMULAS

- **Sine rule:** $a \div \sin A = b \div \sin B = c \div \sin C$
- **Cosine rule:** $a^2 = b^2 + c^2 - 2bc \cdot \cos A$
- **Area rule:** $\text{Area} = \frac{1}{2} \cdot a \cdot b \cdot \sin C$
- Use SINE rule when you have an angle and its OPPOSITE side (AAS or ASA).
- Use COSINE rule when you have NO angle-opposite-side pair (SAS or SSS).

WORKED EXAMPLES

Example 1: Area rule. Find the area of triangle ABC with $a = 8$, $b = 6$, and the included angle $C = 30^\circ$.

- Identify what's given: two sides ($a = 8$, $b = 6$) and the angle BETWEEN them ($C = 30^\circ$).

Two sides and the included angle = use the AREA rule.

- Write the area rule: **Area = $\frac{1}{2} \cdot a \cdot b \cdot \sin C$.**

Multiply the two sides and the sine of the included angle, then halve.

- Substitute: **Area = $\frac{1}{2} \times 8 \times 6 \times \sin 30^\circ$.**

$\sin 30^\circ = 0.5$ (an exact special angle).

- Compute: **Area = $\frac{1}{2} \times 8 \times 6 \times 0.5 = 12$ square units.**

Always include units (e.g. cm^2 or m^2) in the final answer.



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Example 2: Sine rule. In triangle ABC, $A = 70^\circ$, $B = 50^\circ$, and side a (opposite A) = 9. Find side b (opposite B).

- You have an angle and its opposite side (A & a) plus another angle (B). Find that angle's opposite side (b).

Angle-opposite-side pair given $\Rightarrow$ use the SINE rule.

- Write the sine rule: $a \div \sin A = b \div \sin B$.

Pick the two ratios that involve the known and the unknown.

- Substitute and solve for b : $b = a \times (\sin B \div \sin A) = 9 \times (\sin 50^\circ \div \sin 70^\circ)$.

Cross-multiply to isolate b .

- Compute: $b = 9 \times (0.766 \div 0.940) \approx 7.34$.

Use the calculator's sin button — make sure it's in DEGREE mode.

Example 3: Cosine rule. In triangle ABC, $b = 7$, $c = 5$, and the included angle $A = 60^\circ$. Find side a (opposite A).

- Two sides (b , c) and the angle BETWEEN them (A) are given. Find the opposite side a .

SAS configuration with no angle-opposite-side pair $\Rightarrow$ use the COSINE rule.

- Write the cosine rule: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos A$.

The angle is the one OPPOSITE the side you're finding.

- Substitute: $a^2 = 7^2 + 5^2 - 2 \times 7 \times 5 \times \cos 60^\circ$.

$\cos 60^\circ = 0.5$ (an exact special angle).

- Simplify: $a^2 = 49 + 25 - 70 \times 0.5 = 74 - 35 = 39$.

Careful with the order of operations.

- Take the square root: $a = \sqrt{39} \approx 6.24$.

Side lengths are positive — take only the positive root.

Applied example — Area of a plot

A triangular plot has sides 50 m and 70 m with a 60° angle between them. Find its area.

- Area = $\frac{1}{2} \cdot b \cdot c \cdot \sin A = \frac{1}{2} \cdot 50 \cdot 70 \cdot \sin 60^\circ$

Area rule.

- $\approx 1515.54 \text{ m}^2$

Evaluate.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

- Find the area of $\triangle ABC$ with $b = 6$, $c = 8$, $A = 30^\circ$.
- In $\triangle ABC$, $a = 8$, $A = 60^\circ$, $B = 40^\circ$. Use the sine rule to find b .
- In $\triangle ABC$, $b = 7$, $c = 9$, $A = 50^\circ$. Use the cosine rule to find a .
- State the cosine rule for side a .
- Find the area of a triangle with two sides 10 and 12 and included angle 45° (2 dp).
- A triangular plot has sides 50 m and 70 m with a 60° angle between them. Find its area.
- A surveyor measures $\triangle ABC$: $a = 5$, $b = 7$, $c = 8$. Find angle A .
- Two roads leave a point at 70° ; one car drives 8 km, the other 5 km. How far apart are they?
- In $\triangle ABC$, $A = 40^\circ$, $B = 75^\circ$, $a = 10$. Find b .
- A plane flies 200 km on bearing 060° , then 150 km on bearing 120° . How far from the start? (cosine rule, angle 120°)



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End of Chapter 6 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Simplify $\sin^2\theta + \cos^2\theta$.
2. Simplify $\cos(180^\circ - \theta)$.
3. Solve $\tan\theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$.
4. State the cosine rule for side a.
5. Simplify $\sin\theta/\tan\theta$.
6. Simplify $\sin(90^\circ - \theta)$.
7. Solve $\tan\theta = \sqrt{3}$ for $0^\circ \leq \theta \leq 360^\circ$.
8. Two roads leave a point at 70° ; one car drives 8 km, the other 5 km. How far apart are they?
9. Simplify $(\sin^2\theta)/(1 - \cos\theta)$.
10. Evaluate $\cos 300^\circ + \sin 120^\circ$.



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CHAPTER 5

Functions

11.F1 Straight Line function

11.F2 Quadratic function

11.F3 Hyperbolic function

11.F4 Exponential function

11.F1 Straight Line function

DEFINITIONS

Linear function $y = mx + c$ — a straight-line graph.

Gradient m , the steepness of the line.

KEY FORMULAS

- **Standard form:** $y = mx + c$
- **Gradient between two points:** $m = (y_2 - y_1) / (x_2 - x_1)$
- **Point-slope form:** $y - y_1 = m(x - x_1)$
- **Y-intercept:** set $x = 0 \rightarrow y = c$
- **X-intercept:** set $y = 0 \rightarrow x = -c/m$
- **Domain:** $x \in \mathbb{R}$ **Range:** $y \in \mathbb{R}$

WORKED EXAMPLES

Example 1: Find the equation of a line with gradient 2 passing through (1, -2).

► Use point-slope: $y - (-2) = 2(x - 1)$.

Plug into $y - y_1 = m(x - x_1)$.

► Expand: $y + 2 = 2x - 2 \rightarrow y = 2x - 4$.

Simplify into standard form.

Example 2: Find the gradient between (1, 5) and (4, 11).

► $m = (11 - 5) / (4 - 1) = 6/3 = 2$.

Apply the gradient formula.



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Example 3: For $y = 3x - 9$, give the gradient, y-intercept, and x-intercept.

► $m = 3$, $c = -9$.

Read directly.

► **X-intercept:** $0 = 3x - 9 \rightarrow x = 3$.

Solve for x .

Applied example — A taxi-fare graph

A taxi fare is $y = 8x + 12$ (x in km). Find the fare for 9 km and the fixed charge.

► Fixed charge = R12 (y-intercept)

Value when $x = 0$.

► $y = 8(9) + 12 = \text{R}84$

Substitute $x = 9$.

EXERCISES (EASY → DIFFICULT)

- Find the gradient and y-intercept of $y = 3x - 2$.
- Find the x-intercept of $y = 2x - 6$.
- Find the equation of the line through $(0, 1)$ with gradient 2.
- Where do $y = x$ and $y = -x + 4$ intersect?
- Sketch info: state the y-intercept and gradient of $y = -\frac{1}{2}x + 3$.
- A taxi fare is $y = 8x + 12$ (x in km). What is the fixed charge and the rate per km?
- Using $y = 8x + 12$, find the fare for 9 km.
- A candle's height is $h = 20 - 2t$ cm. After how many hours is it fully burnt?
- Find the equation of the line through $(2, 7)$ and $(5, 16)$.
- Two lines $y = 2x + 1$ and $y = -x + 7$ meet where?

11.F2 Quadratic function

DEFINITIONS

Quadratic function

$y = ax^2 + bx + c$ — a parabola.

Turning point

The vertex (maximum or minimum) of the parabola.

KEY FORMULAS

- **Standard form:** $y = ax^2 + bx + c$
- **Quadratic formula:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- **Discriminant:** $\Delta = b^2 - 4ac$ (controls how many real roots)
- **Axis of symmetry:** $x = -b / 2a$
- **Turning point y-coordinate:** $y = c - b^2/(4a)$
- **Domain:** $x \in \mathbb{R}$

WORKED EXAMPLES



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**Example 1: Solve $x^2 - 5x + 6 = 0$.**

► $a = 1$, $b = -5$, $c = 6$. $\Delta = 25 - 24 = 1$.

Discriminant is positive — two real roots.

► $x = (5 \pm 1)/2 \rightarrow x = 3$ or $x = 2$.

Apply the formula.

Example 2: For $y = x^2 - 4x + 3$, find the axis of symmetry and turning point.

► **Axis of symmetry:** $x = -(-4)/(2 \cdot 1) = 2$.

Apply $x = -b/2a$.

► **Turning point y:** $y(2) = 4 - 8 + 3 = -1$. So TP = **(2, -1)**.

Substitute x back in.

Example 3: For $y = x^2 - 6x + 5$, find the roots and turning point.

► Roots: factor as $(x - 1)(x - 5) = 0 \rightarrow x = 1$ or 5 .

Quick factorisation.

► Axis of symmetry $x = -(-6)/2 = 3$. $y(3) = 9 - 18 + 5 = -4$. **TP = (3, -4)**.

Midway between the two roots.

Applied example — Maximum height of a ball

A ball's height is $h = -5t^2 + 20t$. Find its maximum height.

► Axis of symmetry: $t = -b/2a = 20/10 = 2$

Vertex t -value.

► $h(2) = -20 + 40 = 20$ m

Substitute back.

EXERCISES (EASY → DIFFICULT)

1. Find the y -intercept of $y = x^2 - 4x + 3$.
2. Find the x -intercepts of $y = x^2 - 4x + 3$.
3. Find the axis of symmetry of $y = x^2 - 4x + 3$.
4. Find the turning point of $y = x^2 - 4x + 3$.
5. Does $y = x^2 - 4x + 3$ have a minimum or maximum?
6. A ball's height is $h = -5t^2 + 20t$. Find when it reaches the ground again.
7. For $h = -5t^2 + 20t$, find the maximum height (turning point).
8. Find the turning point of $y = x^2 + 6x + 5$.
9. A profit model is $P = -x^2 + 8x - 7$. Find the number of units x giving maximum profit.
10. Find the equation of a parabola with x -intercepts 1 and 5 and passing through (0, 5).

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11.F3 Hyperbolic function

DEFINITIONS

Hyperbola $y = a/x + q$ — a curve with two branches and asymptotes.

Asymptote A line the curve approaches but never touches.

KEY FORMULAS

- **Standard form:** $y = a/(x - h) + k$
- **Vertical asymptote:** $x = h$ (denominator zero)
- **Horizontal asymptote:** $y = k$ (fraction $\rightarrow 0$ at infinity)
- **Centre:** (h, k) — intersection of asymptotes
- **Y-intercept:** set $x = 0 \rightarrow y = a/(-h) + k$
- **Domain:** $x \neq h$ **Range:** $y \neq k$

WORKED EXAMPLES

Example 1: For $y = 2/(x - 1) + 3$, find both asymptotes.

- **Vertical asymptote:** $x - 1 = 0 \rightarrow x = 1$.

Where the denominator is zero.

- **Horizontal asymptote:** $y = 3$.

The constant added outside.

Example 2: For $y = 4/(x + 2) + 1$, find the centre.

- h is the value subtracted: $x + 2 = x - (-2)$ so $h = -2$.

Be careful with signs.

- $k = 1$. Centre is **$(-2, 1)$** .

Where the asymptotes cross.

Example 3: For $y = -3/(x - 2) - 1$, find the y-intercept.

- Set $x = 0$: $y = -3/(0 - 2) - 1 = -3/-2 - 1 = 1.5 - 1 = 0.5$.

Substitute and simplify.

Applied example — Workers and time

A job takes $y = 240/x$ hours for x workers. How many workers finish it in 10 hours?

- $10 = 240/x$

Substitute $y = 10$.

- $x = 24$ workers

Solve the hyperbola equation.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. State the asymptotes of $y = 1/x$.
2. State the asymptotes of $y = 1/x + 2$.
3. State the asymptotes of $y = 2/(x - 3)$.
4. Find y when $x = 2$ for $y = 6/x$.
5. Find x when $y = 2$ for $y = 8/x$.
6. State the asymptotes of $y = 3/(x + 1) - 2$.
7. A job takes $y = 240/x$ hours for x workers. How long do 8 workers take?



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8. Using $y = 240/x$, how many workers finish in 10 hours?
9. Find where $y = 4/x$ meets $y = x$.
10. Write the equation of a hyperbola $y = a/x$ passing through (3, 4).

11.F4 Exponential function

DEFINITIONS

Exponential function $y = a \cdot b^x + q$ with a horizontal asymptote.

Asymptote The value the curve levels off towards.

KEY FORMULAS

- **Standard form:** $y = b^x$, $b > 0$, $b \neq 1$
- **Y-intercept:** always (0, 1)
- **Asymptote:** $y = 0$
- **Equal-base rule:** $b^x = b^k \implies x = k$
- **Domain:** $x \in \mathbb{R}$ **Range:** $y > 0$

WORKED EXAMPLES

Example 1: Solve $2^x = 8$.

► Rewrite: $8 = 2^3$.

Express the right side with the same base.

► $2^x = 2^3 \implies x = 3$.

Apply the equal-base rule.

Example 2: Solve $3^x = 27$.

► $27 = 3^3$.

Common cube.

► $x = 3$.

Equal-base rule.

Example 3: For $y = 2^x$, find the values at $x = -2, 0, 2$.

► $x = -2$: $y = 2^{-2} = 1/4$.

Negative exponent $\rightarrow$ reciprocal.

► $x = 0$: $y = 1$. $x = 2$: $y = 4$.

Y-intercept and a positive power.

► Each time x increases by 2, y is multiplied by $4 = 2^2$.

Multiplicative growth.

Applied example — A depreciating car

A car's value is $V = 200000(0.85)^t$. Find its value after 2 years.

► $V = 200000(0.85)^2$

Substitute $t = 2$.

► = R144,500.00

Evaluate.



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**EXERCISES (EASY → DIFFICULT)**

1. State the asymptote of $y = 2^x$.
2. Find y when $x = 3$ for $y = 2^x$.
3. State the asymptote of $y = 3^x - 4$.
4. Find the y -intercept of $y = 2^x$.
5. Is $y = (\frac{1}{2})^x$ increasing or decreasing?
6. A colony is $N = 2^t$ (t hours). Find the population after 5 hours.
7. A R1 000 investment grows as $A = 1000(1.08)^t$. Find its value after 3 years (2 dp).
8. A car worth R200 000 decays as $V = 200000(0.85)^t$. Find its value after 2 years.
9. Find the y -intercept and asymptote of $y = 3 \cdot 2^x$.
10. A medicine in the body is $M = 50(0.5)^t$ mg (t hours). How much remains after 3 hours?

End of Chapter 5 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Find the gradient and y -intercept of $y = 3x - 2$.
2. Find the x -intercepts of $y = x^2 - 4x + 3$.
3. State the asymptotes of $y = 2/(x - 3)$.
4. Find the y -intercept of $y = 2^x$.
5. Sketch info: state the y -intercept and gradient of $y = -\frac{1}{2}x + 3$.
6. A ball's height is $h = -5t^2 + 20t$. Find when it reaches the ground again.
7. A job takes $y = 240/x$ hours for x workers. How long do 8 workers take?
8. A car worth R200 000 decays as $V = 200000(0.85)^t$. Find its value after 2 years.
9. Find the equation of the line through (2, 7) and (5, 16).
10. Find the equation of a parabola with x -intercepts 1 and 5 and passing through (0, 5).





CHAPTER 1

Exponents

11.E1 Rational exponents and surds

11.E2 Solving surd equations

11.E3 Applications of exponentials

11.E1 Rational exponents and surds

DEFINITIONS

Rational exponent

$a^{(m/n)}$ means the n -th root of a^m .

Surd

A root that cannot be simplified to a rational number, e.g. $\sqrt{2}$.

KEY FORMULAS

- **Surd:** any irrational root like $\sqrt{2}$, $\sqrt{3}$, $\sqrt[3]{5}$
- **Surd product:** $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$
- **Surd quotient:** $\sqrt{a} / \sqrt{b} = \sqrt{a/b}$
- **Simplify by splitting:** $\sqrt{p^2 \cdot q} = p \cdot \sqrt{q}$ (pull out perfect squares)
- **Rationalising the denominator:** $a/\sqrt{b} \times \sqrt{b}/\sqrt{b} = a\sqrt{b}/b$
- **Rational exponent $\leftrightarrow$ surd:** $a^{m/n} = \sqrt[n]{a^m}$

WORKED EXAMPLES

Example 1: Simplify $\sqrt{72}$.

- **Find largest perfect-square factor of 72:** $36 = 6^2$.

$$72 = 36 \times 2.$$

► $\sqrt{72} = \sqrt{(36 \times 2)} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}.$

Pull out $\sqrt{36} = 6$.

Example 2: Simplify $x^{2/3} \times x^{1/3}$.

- **Same base.** Add exponents: $2/3 + 1/3 = 3/3 = 1.$

Apply product rule.

► Result: $x^1 = x.$

Anything to the first power is itself.



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**Example 3: Rationalise the denominator: $3/\sqrt{2}$.**

- **Multiply top and bottom by $\sqrt{2}$.**

Trick: this is just multiplying by 1.

- Numerator: $3 \times \sqrt{2} = 3\sqrt{2}$. Denominator: $\sqrt{2} \times \sqrt{2} = 2$.

Surd $\times$ itself = the radicand.

- Result: $3\sqrt{2} / 2$.

Clean rational denominator.

Applied example — A rational exponent

Simplify $27^{(2/3)}$.

- $27^{(1/3)} = 3$

Cube root first.

- $3^2 = 9$

Then square.

EXERCISES (EASY → DIFFICULT)

1. Simplify $\sqrt{50}$.
2. Simplify $8^{(2/3)}$.
3. Simplify $16^{(3/4)}$.
4. Simplify $\sqrt{12} + \sqrt{27}$.
5. Write $\sqrt{x}$ as a power of x .
6. Simplify $27^{(2/3)}$.
7. Rationalise $1/\sqrt{5}$.
8. Rationalise $1/(2 + \sqrt{3})$.
9. Simplify $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$.
10. Simplify $32^{(2/5)} \times 4^{(1/2)}$.

11.E2 Solving surd equations**DEFINITIONS**

Surd equation An equation with the unknown under a root sign.

Squaring Both sides are squared to remove the root — then check for extraneous solutions.

KEY FORMULAS

- **Strategy:** isolate the surd, then square both sides
- **Squaring rule:** $(\sqrt{A})^2 = A$ (provided $A \geq 0$)
- $(\sqrt{A})^2 = A$ removes the surd
- **Always check answers** by substituting back — squaring can introduce extraneous solutions
- **Restriction:** the expression under the surd must be ≥ 0

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WORKED EXAMPLES

Example 1: Solve $\sqrt{x} = 7$.

► **Square both sides:** $(\sqrt{x})^2 = 7^2$.

Removes the surd.

► **$x = 49$.** Check: $\sqrt{49} = 7$.

Always verify.

Example 2: Solve $\sqrt{x + 5} = 4$.

► **Square both sides:** $x + 5 = 16$.

Removes the surd.

► **$x = 16 - 5 = 11$.** Check: $\sqrt{16} = 4$.

Subtract 5.

Example 3: Solve $\sqrt{3x - 2} = 5$.

► **Square:** $3x - 2 = 25$.

Removes the surd.

► **$3x = 27 \Rightarrow x = 9$.** Check: $\sqrt{(27 - 2)} = \sqrt{25} = 5$.

Solve linearly and verify.

Applied example — A surd equation

Solve $\sqrt{x + 4} = x - 2$, checking for extraneous roots.

► **$x + 4 = (x - 2)^2 \rightarrow x^2 - 5x = 0$**

Square both sides.

► **$x = 0$ or 5 ; $x = 0$ gives $2 = -2$ (false)**

Test each root.

► **$x = 5$**

The valid solution.

EXERCISES (EASY → DIFFICULT)

1. Solve $\sqrt{x} = 5$.
2. Solve $\sqrt{x} = 7$.
3. Solve $\sqrt{x + 1} = 3$.
4. Solve $\sqrt{2x - 1} = 5$.
5. Solve $\sqrt{x} + 2 = 6$.
6. Solve $\sqrt{x + 4} = x - 2$ (check for extraneous roots).
7. Why is $x = 0$ rejected in $\sqrt{x + 4} = x - 2$?
8. Solve $\sqrt{3x + 1} = x - 1$.
9. Solve $x^{2/3} = 4$.
10. Solve $\sqrt{x - 3} = \sqrt{2x - 7}$.

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11.E3 Applications of exponentials (growth/decay)

DEFINITIONS

Exponential growth

A quantity multiplied by a constant factor each period.

Exponential model

$y = a \cdot b^x$ describes growth ($b > 1$) or decay ($0 < b < 1$).

KEY FORMULAS

- **Population growth:** $N = N_0 \times (1 + r)^t$
- **Compound interest:** $A = P \times (1 + r)^t$
- **Depreciation (straight-line):** $V = P \times (1 - r \cdot t)$
- **Depreciation (reducing balance):** $V = P \times (1 - r)^t$
- **Bacterial doubling:** $N = N_0 \times 2^t$
- **r as a decimal:** 10% = 0.10, 20% = 0.20

WORKED EXAMPLES

Example 1: Bacteria doubles every hour. Start with 200. How many after 3 hours?

► **Formula:** $N = N_0 \times 2^t$.

Doubling each hour.

► $2^3 = 8$. So $N = 200 \times 8 = \mathbf{1\ 600}$.

Multiply.

Example 2: R5 000 invested at 10% compound interest. Value after 2 years?

► **Formula:** $A = P(1 + r)^t$. Here $P = 5\ 000$, $r = 0.10$, $t = 2$.

Convert percent to decimal.

► $(1.10)^2 = 1.21$.

Square the multiplier.

► $A = 5\ 000 \times 1.21 = \mathbf{R6\ 050}$.

Multiply.

Example 3: A car worth R200 000 depreciates 20% per year (reducing balance). Value after 2 years?

► **Formula:** $V = P(1 - r)^t$. Here $r = 0.20$, so multiplier = 0.80.

Subtract for decay.

► $(0.80)^2 = 0.64$.

Square the multiplier.

► $V = 200\ 000 \times 0.64 = \mathbf{R128\ 000}$.

Multiply.

Applied example — Population growth

A town of 20 000 grows 4% a year: $P = 20000(1.04)^t$. Find P after 5 years.

► $P = 20000(1.04)^5$

Substitute $t = 5$.

► ≈ 24333 people

Evaluate and round.



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**EXERCISES (EASY → DIFFICULT)**

1. A colony doubles each hour: $N = 100 \cdot 2^t$. Find N after 3 hours.
2. A R5 000 investment grows as $A = 5000(1.1)^t$. Find A after 2 years.
3. A car decays as $V = 180000(0.8)^t$. Find its value after 3 years.
4. A medicine halves each hour: $M = 80(0.5)^t$ mg. Find M after 4 hours.
5. Bacteria triple every hour from 50: $N = 50 \cdot 3^t$. Find N after 2 hours.
6. A town's population grows 4% a year from 20 000: $P = 20000(1.04)^t$. Find P after 5 years (nearest person).
7. A R12 000 laptop loses 25% of its value each year. Find its worth after 3 years.
8. A colony $N = 100 \cdot 2^t$ reaches 1 600 after how many hours?
9. An investment doubles in value: $5000 \cdot (1.09)^t = 10000$. Find t (2 dp).
10. A radioactive sample halves every 5 years. What fraction remains after 15 years?

End of Chapter 1 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Simplify $\sqrt{50}$.
2. Solve $\sqrt{x} = 7$.
3. A car decays as $V = 180000(0.8)^t$. Find its value after 3 years.
4. Simplify $\sqrt{12} + \sqrt{27}$.
5. Solve $\sqrt{x} + 2 = 6$.
6. A town's population grows 4% a year from 20 000: $P = 20000(1.04)^t$. Find P after 5 years (nearest person).
7. Rationalise $1/\sqrt{5}$.
8. Solve $\sqrt{(3x + 1)} = x - 1$.
9. An investment doubles in value: $5000 \cdot (1.09)^t = 10000$. Find t (2 dp).
10. Simplify $32^{(2/5)} \times 4^{(1/2)}$.



**CHAPTER 3**

Number Patterns

11.P1 Arithmetic sequences**11.P2** Quadratic sequences**11.P3** Geometric sequences

11.P1 Arithmetic sequences

**DEFINITIONS****Arithmetic sequence**A sequence with a constant difference d between terms.**Common difference**

$$d = T_n - T_{n-1}.$$

KEY FORMULAS

- **Arithmetic sequence:** common difference d is constant
- **Common difference:** $d = T_{n+1} - T_n$
- **n -th term:** $T_n = a + (n - 1)d$
- **Solve for n given T :** $n = (T - a)/d + 1$
- **Two unknowns?** Use two equations and solve simultaneously

WORKED EXAMPLES**Example 1: For 2, 9, 16, 23, ... find T_5 .**

► $a = 2, d = 9 - 2 = 7.$

Read off.

► $T_5 = 2 + 4 \times 7 = 2 + 28 = 30.$

Apply formula.

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**Example 2: An arithmetic sequence has $T_2 = 9$ and $T_5 = 24$. Find a and d .**

► **Set up equations:** $a + d = 9$, $a + 4d = 24$.

Use $T_n = a + (n - 1)d$ for $n = 2$ and $n = 5$.

► **Subtract:** $3d = 15 \Rightarrow d = 5$. Then $a = 9 - 5 = 4$.

Solve simultaneously.

Example 3: For 5, 9, 13, ..., which term equals 49?

► $a = 5$, $d = 4$. Set $T_n = 49$.

We need to find n .

► $5 + (n - 1) \times 4 = 49 \Rightarrow (n - 1) \times 4 = 44 \Rightarrow n - 1 = 11 \Rightarrow n = 12$.

Standard algebra.

Applied example — Seats in a theatre

A theatre has 20 seats in row 1 and 4 more in each next row. How many seats are in row 10?

► $a = 20$, $d = 4$ (arithmetic)

Constant difference.

► $T_{10} = 20 + 9(4) = 56$ seats

$T_n = a + (n - 1)d$.

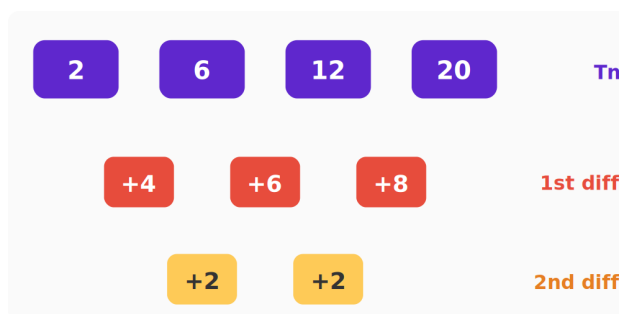
EXERCISES (EASY → DIFFICULT)

1. Find the next term of 3, 7, 11, 15, ...
2. Find the common difference of 5, 9, 13, ...
3. For 2, 5, 8, ... find T_{10} .
4. Find T_n for 1, 6, 11, 16, ...
5. Find the sum of the first 20 terms of 3, 7, 11, ...
6. A theatre has 20 seats in row 1 and 4 more in each next row. How many seats in row 10?
7. Lerato's salary starts at R8 000 and rises R500 each year. What is her salary in year 6?
8. Logs are stacked 3 in the top row, then 4 more each row. How many logs in the first 12 rows?
9. Which term of 4, 7, 10, ... equals 100?
10. The 3rd term of an arithmetic sequence is 11 and the 7th is 27. Find a and d .

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11.P2 Quadratic sequences



DEFINITIONS

Quadratic sequence

A sequence whose second differences are constant.

Second difference

The difference of the differences between terms.

KEY FORMULAS

- **Quadratic sequence:** the **SECOND** difference is constant (not the first)
- **General term:** $T_n = an^2 + bn + c$
- **Finding a:** $2a =$ the constant second difference
- **Finding b and c:** use T_1 and T_2 to set up equations
- **Spotting:** compute first differences. If they're not constant, compute **SECOND** differences.

WORKED EXAMPLES

Example 1: Find a, b, c for the sequence 2, 6, 12, 20.

- **1st differences:** 4, 6, 8.

Subtract consecutive terms.

- **2nd differences:** 2, 2 — constant. So $2a = 2 \Rightarrow a = 1$.

Confirms quadratic; gives a.

- $T_1 = a + b + c = 2 \Rightarrow b + c = 1$. $T_2 = 4a + 2b + c = 6 \Rightarrow 2b + c = 2$.

Two equations from T_1 and T_2 .

- Subtract: $b = 1$, then $c = 0$. Formula: $T_n = n^2 + n$.

Solve simultaneously.

Example 2: Find a for 1, 4, 9, 16.

- 1st differences: 3, 5, 7.

Compute.

- 2nd differences: 2, 2. So $a = 1$.

Constant 2nd difference is 2, so $a = 2/2 = 1$.

Example 3: Find a for 5, 11, 21, 35.

- 1st differences: 6, 10, 14.

Compute.

- 2nd differences: 4, 4 — constant. So $2a = 4 \Rightarrow a = 2$.

Bigger 2nd difference means a bigger leading coefficient.



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**Applied example — A growing dot pattern**

A dot pattern grows 2, 6, 12, 20, ... Find a formula T_n .

- First diffs 4, 6, 8; second diff 2 $\Rightarrow a = 1$

Quadratic sequence ($2a = 2$).

- $T_n = n^2 + n$

Fit the first terms.

EXERCISES (EASY → DIFFICULT)

- Find the second difference of 1, 4, 9, 16, ...
- Find a (where $T_n = an^2 + bn + c$) for 1, 4, 9, 16.
- Find the next term of 3, 8, 15, 24, ...
- Find T_n for 1, 4, 9, 16, ...
- Find T_n for 2, 6, 12, 20, ...
- A dot pattern grows 1, 4, 9, 16, ... How many dots are in the 10th figure?
- Bricks form a pattern 2, 6, 12, 20, ... per layer. Find a formula T_n .
- A quadratic sequence has first differences 4, 7, 10, ... and $T_1 = 2$. Find T_4 .
- For 3, 8, 17, 30, ... find T_5 .
- For $T_n = n^2 - 5n + 6$, find n where $T_n = 0$.

11.P3 Geometric sequences**DEFINITIONS****Geometric sequence**

A sequence with a constant ratio r between terms.

Common ratio

$$r = T_n / T_{n-1}.$$

KEY FORMULAS

- **Geometric sequence:** each term multiplies by a fixed common ratio r
- **Common ratio:** $r = T_{n+1} / T_n$
- **n -th term:** $T_n = a \cdot r^{n-1}$
- **Spotting:** dividing consecutive terms always gives the same number
- **Negative r** means terms alternate sign

WORKED EXAMPLES**DIGITAL TEXTBOOK**

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**Example 1: For 2, 6, 18, 54, ... find T_5 .**

► $a = 2$, $r = 6/2 = 3$.

Read off and divide.

► $T_5 = 2 \cdot 3^4 = 2 \cdot 81 = 162$.

Plug $n=5$ into the formula.

Example 2: For 3, 6, 12, 24, ... find T_6 .

► $a = 3$, $r = 6/3 = 2$.

Read off and divide.

► $T_6 = 3 \cdot 2^5 = 3 \cdot 32 = 96$.

Plug $n=6$.

Example 3: For 16, 8, 4, 2, ... find T_5 (decreasing).

► $a = 16$, $r = 8/16 = 1/2$.

Ratio less than 1 $\rightarrow$ decreasing.

► $T_5 = 16 \cdot (1/2)^4 = 16 \cdot 1/16 = 1$.

Multiplying by $1/2$ four times.

Applied example — A bouncing ball

A ball bounces to half its height each time: 16, 8, 4, ... m. How high is the 6th bounce?

► $a = 16$, $r = 1/2$ (geometric)

Constant ratio.

► $T_6 = 16(1/2)^5 = 0.5$ m

$T_n = a \cdot r^{n-1}$.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. Find the next term of 2, 6, 18, ...
2. Find the common ratio of 3, 6, 12, ...
3. For 1, 5, 25, ... find T_5 .
4. For 3, 6, 12, ... find T_7 .
5. Find T_n for 4, 8, 16, ...
6. A ball bounces to half its height each time: 16, 8, 4, ... m. How high is the 6th bounce?
7. A rumour doubles the listeners each round, starting at 3. Which round reaches 96? (3, 6, 12, ...)
8. A car worth R200 000 keeps 80% of its value yearly: 200 000, 160 000, ... Find its value after 3 years (4th term).
9. The 2nd term of a geometric sequence is 6 and the 4th is 54 ($r > 0$). Find r .
10. Three numbers x , $x + 3$, $x + 9$ are geometric. Find x .

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End of Chapter 3 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Find the next term of 3, 7, 11, 15, ...
2. Find a (where $T_n = an^2 + bn + c$) for 1, 4, 9, 16.
3. For 1, 5, 25, ... find T_5 .
4. Find T_n for 1, 6, 11, 16, ...
5. Find T_n for 2, 6, 12, 20, ...
6. A ball bounces to half its height each time: 16, 8, 4, ... m. How high is the 6th bounce?
7. Lerato's salary starts at R8 000 and rises R500 each year. What is her salary in year 6?
8. A quadratic sequence has first differences 4, 7, 10, ... and $T_1 = 2$. Find T_4 .
9. The 2nd term of a geometric sequence is 6 and the 4th is 54 ($r > 0$). Find r .
10. The 3rd term of an arithmetic sequence is 11 and the 7th is 27. Find a and d .



ANSWERS

Worked solutions to all exercises

Each answer gives the final result first, followed by the key working steps. For fully animated, step-by-step solutions, open the digital edition with the QR code on any page.

CHAPTER 2 — Equations and inequalities

2.1 Revision: solving quadratics by factorising

- $(x - 2)(x - 3) = 0 \rightarrow x = 2 \text{ or } 3.$
- $x = \pm 3.$
- $(x + 4)(x - 2) = 0 \rightarrow x = -4 \text{ or } 2.$
- $(x - 3)(x - 4) = 0 \rightarrow x = 3 \text{ or } 4.$
- $(2x + 1)(x - 3) = 0 \rightarrow x = -\frac{1}{2} \text{ or } 3.$
- $n(n + 1) = 56 \rightarrow n^2 + n - 56 = 0 \rightarrow (n + 8)(n - 7) = 0 \rightarrow 7 \text{ and } 8 \text{ (or } -8 \text{ and } -7).$
- $x^2 - x = 12 \rightarrow (x - 4)(x + 3) = 0 \rightarrow x = 4 \text{ or } -3.$
- $w(w + 3) = 40 \rightarrow w^2 + 3w - 40 = 0 \rightarrow (w + 8)(w - 5) = 0 \rightarrow w = 5 \text{ m.}$
- $w(w + 5) = 36 \rightarrow (w + 9)(w - 4) = 0 \rightarrow w = 4 \text{ cm.}$
- $\frac{1}{2}h \cdot (h + 2) = 24 \rightarrow h^2 + 2h - 48 = 0 \rightarrow (h + 8)(h - 6) = 0 \rightarrow h = 6 \text{ cm.}$

2.2 Completing the square

- Add $(6/2)^2 = 9 \rightarrow x^2 + 6x + 9 = (x + 3)^2.$
- Add 25 $\rightarrow (x - 5)^2.$
- $(x + 2)^2 = 9 \rightarrow x + 2 = \pm 3 \rightarrow x = 1 \text{ or } -5.$
- $(x - 1)^2 = 4 \rightarrow x = 3 \text{ or } -1.$
- $(x - 2)^2 = 3 \rightarrow x = 2 \pm \sqrt{3}.$
- $(x - 3)^2 - 4 \rightarrow$ turning point $(3, -4).$
- $(x + 1)^2 + 4 \rightarrow$ minimum value 4.
- $h = -(t - 2)^2 + 4 \rightarrow$ maximum height 4 m at $t = 2 \text{ s.}$
- $P = -(x - 5)^2 + 16 \rightarrow$ maximum profit 16 at $x = 5.$
- $x^2 - 4x + 1.5 = 0 \rightarrow (x - 2)^2 = 2.5 \rightarrow x = 2 \pm \sqrt{2.5} \approx 3.58 \text{ or } 0.42.$

2.3 Quadratic formula

- $(x - 4)(x + 1) = 0 \rightarrow x = 4 \text{ or } -1.$
- $x = 2 \text{ or } -3.$
- $x = 3 \text{ or } \frac{1}{2}.$
- $x \approx 3.73 \text{ or } 0.27.$
- $x \approx 1.22 \text{ or } -0.55.$
- $5t^2 - 25t + 20 = 0 \rightarrow t^2 - 5t + 4 = 0 \rightarrow t = 1 \text{ s or } 4 \text{ s.}$
- $w^2 + 3w - 50 = 0 \rightarrow w \approx 5.73 \text{ m.}$
- $x = 3 \pm \sqrt{2}.$
- $x \approx 0.85 \text{ or } -2.35.$
- $(8 + 2x)(10 + 2x) = 143 \rightarrow 4x^2 + 36x - 63 = 0 \rightarrow x = 1.5 \text{ cm.}$



2.4 Substitution (k-method)

- $k^2 - 5k + 4 = 0 \rightarrow k = 1 \text{ or } 4 \rightarrow x = \pm 1, \pm 2.$
- $k = x^2: k = 4 \text{ or } 9 \rightarrow x = \pm 2, \pm 3.$
- $k = 1 \text{ or } 9 \rightarrow x = \pm 1, \pm 3.$
- $2k^2 - 9k + 4 = 0 \rightarrow k = 4 \text{ or } \frac{1}{2} \rightarrow x = \pm 2, \pm 1/\sqrt{2}.$
- $k^2 - 8k + 12 = 0 \rightarrow k = 2 \text{ or } 6 \rightarrow x^2 - x - 2 = 0 \text{ or } x^2 - x - 6 = 0 \rightarrow x = 2, -1, 3, -2.$
- $k^2 - 7k + 12 = 0 \rightarrow k = 3 \text{ or } 4 \rightarrow \sqrt{x} = 3 \text{ or } 4 \rightarrow x = 9 \text{ or } 16.$
- $k^2 + k - 6 = 0 \rightarrow k = 2 \text{ (reject } -3) \rightarrow \sqrt{x} = 2 \rightarrow x = 4.$
- $k = 2 \text{ or } 3 \rightarrow 2x - 1 = 2 \text{ or } 3 \rightarrow x = 3/2 \text{ or } 2.$
- $k^2 - 5k + 4 = 0 \rightarrow k = 1 \text{ or } 4 \rightarrow x = 0 \text{ or } 2.$
- $k = 1 \text{ or } 3; x + 1/x = 1 \text{ has no real solution, } x + 1/x = 3 \rightarrow x^2 - 3x + 1 = 0 \rightarrow x = (3 \pm \sqrt{5})/2.$

2.5 Finding the equation given roots

- $x^2 - (2+3)x + 2 \cdot 3 = 0 \rightarrow x^2 - 5x + 6 = 0.$
- sum 3, product $-4 \rightarrow x^2 - 3x - 4 = 0.$
- sum -7 , product $10 \rightarrow x^2 + 7x + 10 = 0.$
- $x^2 - 6x = 0.$
- $(x - 3)^2 = x^2 - 6x + 9 = 0.$
- $x^2 - 7x + 10 = 0.$
- sum 2, product $1 - 2 = -1 \rightarrow x^2 - 2x - 1 = 0.$
- sum $-5/2$, product $-3/2 \rightarrow \times 2: 2x^2 + 5x - 3 = 0.$
- product = 12 $\rightarrow$ other root 4; sum = 7 $\rightarrow b = -7.$
- roots $1/2, 1/3 \rightarrow$ sum $5/6$, product $1/6 \rightarrow \times 6: 6x^2 - 5x + 1 = 0.$

2.6 Nature of roots (discriminant)

- $\Delta = b^2 - 4ac.$
- $\Delta = 0 \rightarrow$ roots are **real and equal**.
- $\Delta = -3 \rightarrow$ roots are **non-real**.
- $\Delta = 1 \rightarrow$ **real, unequal, rational**.
- $\Delta = 17 \rightarrow$ **real, unequal, irrational**.
- $k^2 - 36 = 0 \rightarrow k = \pm 6.$
- $16 - 4k \geq 0 \rightarrow k \leq 4.$
- $\Delta = 4 - 12 = -8 < 0 \rightarrow$ **non-real** (no real roots).
- $16 - 8k < 0 \rightarrow k > 2.$
- $\Delta = (m + 1)^2 - 4m = (m - 1)^2 \geq 0 \text{ for all } m \rightarrow$ **always real**.

2.7 Quadratic inequalities

- $x > 3.$
- $x < 1 \text{ or } x > 4.$
- $-2 < x < 3.$
- $-3 \leq x \leq 3.$
- $(x - 2)(x - 3) > 0 \rightarrow x < 2 \text{ or } x > 3.$
- $x < 2 \text{ or } x > 6.$
- $x(4 - x) > 0 \rightarrow 0 < x < 4.$
- $x(x - 4) \geq 0 \rightarrow x \leq 0 \text{ or } x \geq 4.$
- $(x - 4)(x + 2) \leq 0 \rightarrow -2 \leq x \leq 4.$
- The square is ≥ 0 , so sign follows $(x + 3): x > -3, x \neq 1.$

2.8 Simultaneous (1 linear + 1 quadratic)

- $x^2 = x + 1 \rightarrow x^2 - x - 1 = 0 \rightarrow x = (1 \pm \sqrt{5})/2; \text{ then } y = x + 1.$
- $x^2 - 2x - 3 = 0 \rightarrow x = 3 \text{ or } -1 \rightarrow (3, 6) \text{ and } (-1, -2).$
- $x^2 - 4x + 3 = 0 \rightarrow x = 1 \text{ or } 3 \rightarrow (1, 0) \text{ and } (3, 2).$
- $y = 5 - x \rightarrow x^2 - 5x + 6 = 0 \rightarrow x = 2 \text{ or } 3 \rightarrow (2, 3) \text{ and } (3, 2).$
- $x^2 + (x + 2)^2 = 10 \rightarrow 2x^2 + 4x - 6 = 0 \rightarrow x = 1 \text{ or } -3 \rightarrow (1, 3) \text{ and } (-3, -1).$
- $x + y = 10, xy = 21 \rightarrow x^2 - 10x + 21 = 0 \rightarrow 3 \text{ and } 7.$





7. $x^2 - 2x - 3 = 0 \rightarrow x = 3$ or $-1 \rightarrow$ **(3, 4) and (-1, 0).**
8. $x(x - 4) = 45 \rightarrow x^2 - 4x - 45 = 0 \rightarrow x = 9$ or $-5 \rightarrow$ **9 and 5** (or -5 and -9).
9. $x^2 - 2x - 1 = 0 \rightarrow x = 1 \pm \sqrt{2} \rightarrow$ then $y = 2x - 1$.
10. $x + y = 13, xy = 40 \rightarrow x^2 - 13x + 40 = 0 \rightarrow$ **5 m $\times$ 8 m.**

2.9 Word problems (quadratic)

1. $n(n + 2) = 80 \rightarrow n^2 + 2n - 80 = 0 \rightarrow (n + 10)(n - 8) = 0 \rightarrow$ **8 and 10.**
2. $x^2 + x - 30 = 0 \rightarrow (x + 6)(x - 5) = 0 \rightarrow$ **x = 5.**
3. $w(w + 5) = 84 \rightarrow (w + 12)(w - 7) = 0 \rightarrow$ **7 m $\times$ 12 m.**
4. $n^2 + (n + 1)^2 = 25 \rightarrow 2n^2 + 2n - 24 = 0 \rightarrow n = 3$ or $-4 \rightarrow$ **3 and 4** (or -4 and -3).
5. $5t(6 - t) = 0 \rightarrow$ **t = 6 s.**
6. $s(s + 3) = 70 \rightarrow s^2 + 3s - 70 = 0 \rightarrow (s + 10)(s - 7) = 0 \rightarrow$ **Sipho is 7.**
7. $(6 + 2)(4 + 2) =$ **48 m².**
8. $(10 + 2x)(6 + 2x) = 96 \rightarrow 4x^2 + 32x - 36 = 0 \rightarrow x^2 + 8x - 9 = 0 \rightarrow$ **x = 1 m.**
9. $5t^2 = 45 \rightarrow t^2 = 9 \rightarrow$ **t = 3 s.**
10. $120/n - 120/(n+2) = 2 \rightarrow n^2 + 2n - 120 = 0 \rightarrow (n + 12)(n - 10) = 0 \rightarrow$ **10 learners.**

End of Chapter 2 — Mixed Exercise

1. $(x - 2)(x - 3) = 0 \rightarrow$ **x = 2 or 3.**
2. Add 25 $\rightarrow$ **(x - 5)².**
3. **x = 3 or $\frac{1}{2}$.**
4. $2k^2 - 9k + 4 = 0 \rightarrow k = 4$ or $\frac{1}{2} \rightarrow$ **x = $\pm 2, \pm 1/\sqrt{2}$.**
5. **(x - 3)² = x² - 6x + 9 = 0.**
6. $k^2 - 36 = 0 \rightarrow$ **k = ± 6 .**
7. $x(4 - x) > 0 \rightarrow$ **0 < x < 4.**
8. $x(x - 4) = 45 \rightarrow x^2 - 4x - 45 = 0 \rightarrow x = 9$ or $-5 \rightarrow$ **9 and 5** (or -5 and -9).
9. $5t^2 = 45 \rightarrow t^2 = 9 \rightarrow$ **t = 3 s.**
10. $\frac{1}{2} \cdot h \cdot (h + 2) = 24 \rightarrow h^2 + 2h - 48 = 0 \rightarrow (h + 8)(h - 6) = 0 \rightarrow$ **h = 6 cm.**

CHAPTER 4 — Analytical geometry

4.1 Revision: distance, midpoint, gradient

1. $\sqrt{9 + 16} = 5$.
2. **(4, 6).**
3. $(8 - 2)/(3 - 1) = 3$.
4. **5.**
5. **(1, 1).**
6. **5 km.**
7. $(9 - 1)/(5 - 1) = 2$.
8. **(2, -1).**
9. **5 km.**
10. Gradient $(1,2) \rightarrow (4,3) = 1/3$; $(4,3) \rightarrow (7,4) = 1/3 \rightarrow$ equal gradients, so **collinear.**

4.2 Equation of a line

1. **y = 2x - 3.**
2. **y = -x + 4.**
3. $y - 2 = 3(x - 1) \rightarrow$ **y = 3x - 1.**
4. $m = 2 \rightarrow y - 5 = 2(x - 2) \rightarrow$ **y = 2x + 1.**
5. **y = 2x.**
6. **y = 8x + 12.**
7. **y = 250x + 350.**
8. $m = 3 \rightarrow y - 4 = 3(x - 1) \rightarrow$ **y = 3x + 1.**
9. x-int (6, 0); y-int (0, 4).
10. $m = (12 - 20)/4 = -2 \rightarrow$ **h = -2x + 20.**





4.3 Inclination of a line

1. $\tan 45^\circ = 1$.
2. 45° .
3. $\tan \theta = \sqrt{3} \rightarrow 60^\circ$.
4. $\tan 30^\circ = 1/\sqrt{3}$.
5. 0° .
6. $\tan \theta = 1/4 \rightarrow \theta \approx 14.04^\circ$.
7. $\tan \theta = 2 \rightarrow \theta \approx 63.43^\circ$.
8. $m = 1 \rightarrow 45^\circ$.
9. $\tan 135^\circ = -1$.
10. $\tan \theta = 5/2 \rightarrow \theta \approx 68.2^\circ$.

4.4 Parallel lines

1. They are equal.
2. Both have gradient 2 $\rightarrow$ **yes**.
3. **3**.
4. **$y = 4x + 1$** .
5. $y - 5 = 2(x - 2) \rightarrow y = 2x + 1$.
6. $m = \frac{1}{2} \rightarrow y - 1 = \frac{1}{2}(x - 4) \rightarrow y = \frac{1}{2}x - 1$.
7. Gradients 2 and 2 $\rightarrow$ **yes, parallel**.
8. $2x - y = 5 \rightarrow y = 2x - 5$, gradient 2 $\rightarrow$ **a = 2**.
9. $y - 2 = -3(x - 1) \rightarrow y = -3x + 5$.
10. Both gradients are 2 $\rightarrow$ those two sides are **parallel**.

4.5 Perpendicular lines

1. -1 .
2. $-\frac{1}{2}$.
3. **3**.
4. $2 \times (-\frac{1}{2}) = -1 \rightarrow$ **yes**.
5. **$y = -\frac{1}{2}x$** .
6. perpendicular gradient $= -\frac{1}{3} \rightarrow y - 9 = -\frac{1}{3}(x - 3) \rightarrow y = -\frac{1}{3}x + 10$.
7. midpoint (3, 4); AB gradient 1 $\rightarrow$ perp gradient $-1 \rightarrow y = -x + 7$.
8. $2k = -1 \rightarrow k = -\frac{1}{2}$.
9. Gradients 2 and $-\frac{1}{2}$; product $-1 \rightarrow$ **perpendicular**.
10. $-1 \div (\frac{3}{4}) = -\frac{4}{3}$.

End of Chapter 4 — Mixed Exercise

1. $\sqrt{9 + 16} = 5$.
2. **$y = -x + 4$** .
3. $\tan \theta = \sqrt{3} \rightarrow 60^\circ$.
4. **$y = 4x + 1$** .
5. **$y = -\frac{1}{2}x$** .
6. **5 km**.
7. **$y = 250x + 350$** .
8. $m = 1 \rightarrow 45^\circ$.
9. $y - 2 = -3(x - 1) \rightarrow y = -3x + 5$.
10. $-1 \div (\frac{3}{4}) = -\frac{4}{3}$.



CHAPTER 7 — Measurement

7.1 Area of polygon

1. 40 cm^2 .
2. $\frac{1}{2} \cdot 10 \cdot 6 = 30 \text{ cm}^2$.
3. 49 cm^2 .
4. $\frac{1}{2}(6 + 10) \cdot 4 = 32 \text{ cm}^2$.
5. $(22/7) \cdot 7^2 = 154 \text{ cm}^2$.
6. 450 m^2 .
7. $\frac{1}{2} \cdot 4 \cdot 6 = 12 \text{ m}^2$.
8. $(22/7) \cdot 0.49 = 1.54 \text{ m}^2$.
9. $\frac{1}{2}(30 + 50) \cdot 20 = 800 \text{ m}^2$.
10. $24 \text{ m}^2 \div 0.25 \text{ m}^2 = 96 \text{ tiles}$.

7.2 Right prism / cylinder volume

1. 60 cm^3 .
2. 125 cm^3 .
3. 160 cm^3 .
4. $(22/7) \cdot 49 \cdot 10 = 1\,540 \text{ cm}^3$.
5. 180 cm^3 .
6. $3 \text{ m}^3 = 3\,000 \text{ L}$.
7. $(22/7)(0.1225)(1) \approx 0.385 \text{ m}^3 = 385 \text{ L}$.
8. $10 \cdot 4 \cdot 1.5 = 60 \text{ m}^3$.
9. $154 \cdot h = 3080 \rightarrow h = 20 \text{ cm}$.
10. $3.14 \cdot 0.04 \cdot 3 \approx 0.38 \text{ m}^3$.

7.3 Volume of right cone ($\pi=3$)

1. $\frac{1}{3} \cdot 3 \cdot 25 \cdot 6 = 150 \text{ cm}^3$.
2. $\frac{1}{3} \cdot 3 \cdot 4 \cdot 9 = 36 \text{ cm}^3$.
3. $\frac{1}{3} \cdot 3 \cdot 100 \cdot 3 = 300 \text{ cm}^3$.
4. $\frac{1}{3} \cdot 3 \cdot 16 \cdot 6 = 96 \text{ cm}^3$.
5. $\frac{1}{3} \cdot 3 \cdot 9 \cdot 10 = 90 \text{ cm}^3$.
6. $\frac{1}{3} \cdot 3 \cdot 9 \cdot 12 = 108 \text{ cm}^3$.
7. $\frac{1}{3} \cdot 3 \cdot 4 \cdot 1.5 = 6 \text{ m}^3$.
8. $\frac{1}{3} \cdot 3 \cdot 25 \cdot 9 = 225 \text{ cm}^3$.
9. $\frac{1}{3} \cdot 3 \cdot 36 \cdot h = 360 \rightarrow 36h = 360 \rightarrow h = 10 \text{ cm}$.
10. $\frac{1}{3} \cdot 3 \cdot 49 \cdot 24 = 1\,176 \text{ cm}^3$.

7.3 Volume of sphere

1. $\frac{4}{3} \cdot 3 \cdot 27 = 108 \text{ cm}^3$.
2. $\frac{4}{3} \cdot 3 \cdot 216 = 864 \text{ cm}^3$.
3. $\frac{4}{3} \cdot 3 \cdot 1 = 4 \text{ cm}^3$.
4. $\frac{4}{3} \cdot 3 \cdot 125 = 500 \text{ cm}^3$.
5. $\frac{4}{3} \cdot 3 \cdot 8 = 32 \text{ cm}^3$.
6. $\frac{4}{3} \cdot 3 \cdot 27 = 108 \text{ cm}^3$.
7. $\frac{4}{3} \cdot 3.14 \cdot 1331 \approx 5572 \text{ cm}^3$.
8. $\frac{4}{3} \cdot 3 \cdot 0.125 = 0.5 \text{ m}^3 = 500 \text{ L}$.
9. each marble $4 \text{ cm}^3 \rightarrow 1 \text{ marble}$.
10. $4r^3 = 256 \rightarrow r^3 = 64 \rightarrow r = 4 \text{ cm}$.





7.4 Multiplying a dimension by k

1. $2^2 = 4$ times.
2. $2^3 = 8$ times.
3. 9 times.
4. 27 times.
5. $(\frac{1}{2})^3 = \frac{1}{8}$.
6. $2^3 = 8$ times (volume scales with the cube of the linear factor).
7. $10^2 = 100$ times.
8. $1.5^3 = 3.375$ times.
9. $12 \times 16 = 192 \text{ cm}^2$.
10. $30k^3 = 810 \rightarrow k^3 = 27 \rightarrow k = 3$.

End of Chapter 7 — Mixed Exercise

1. 40 cm^2 .
2. 125 cm^3 .
3. $\frac{1}{3} \cdot 3 \cdot 100 \cdot 3 = 300 \text{ cm}^3$.
4. $\frac{4}{3} \cdot 3 \cdot 125 = 500 \text{ cm}^3$.
5. $(\frac{1}{2})^3 = \frac{1}{8}$.
6. 450 m^2 .
7. $(\frac{22}{7})(0.1225)(1) \approx 0.385 \text{ m}^3 = 385 \text{ L}$.
8. $\frac{1}{3} \cdot 3 \cdot 25 \cdot 9 = 225 \text{ cm}^3$.
9. each marble $4 \text{ cm}^3 \rightarrow 1$ marble.
10. $30k^3 = 810 \rightarrow k^3 = 27 \rightarrow k = 3$.

CHAPTER 8 — Euclidean geometry — circle theorems

8.1 Revision: angle sum of triangle

1. 50° .
2. 55° .
3. 60° .
4. $6x = 180 \rightarrow x = 30^\circ$.
5. 65° .
6. $(180 - 50)/2 = 65^\circ$.
7. $\frac{4}{9} \cdot 180 = 80^\circ$.
8. $(180 - 70)/2 = 55^\circ$.
9. $4x + 40 = 180 \rightarrow x = 35^\circ$.
10. $90^\circ, 45^\circ, 45^\circ$.

8.2 Tangent $\perp$ radius

1. 90° .
2. Exactly one.
3. $x = 90^\circ$.
4. Perpendicular $\rightarrow -\frac{1}{2}$.
5. $\angle OTP = 90^\circ \rightarrow OP = \sqrt{(25 + 144)} = 13$.
6. 90° .
7. $180 - 90 - 40 = 50^\circ$.
8. $\sqrt{(15^2 - 8^2)} = \sqrt{161} \approx 12.69 \text{ m}$.
9. $\sqrt{(100 - 36)} = 8$.
10. The radius is the shortest distance from the centre to the tangent line, which occurs at a right angle.





8.2 Perpendicular from centre bisects chord

1. Bisects it (cuts it in half).
2. **8 cm.**
3. $\sqrt{(100 - 36)} = 8$, so the chord is 16.
4. half-chord 12 $\rightarrow \sqrt{(169 - 144)} = 5$.
5. half-chord 4 $\rightarrow \sqrt{(25 - 16)} = 3$.
6. half $= \sqrt{(25^2 - 14^2)} = \sqrt{429} \approx 20.71 \rightarrow \text{chord} \approx \mathbf{41.42 \text{ cm.}}$
7. half $= \sqrt{(169 - 25)} = 12 \rightarrow \text{beam} = \mathbf{24 \text{ m.}}$
8. half-chord 15 $\rightarrow \sqrt{(289 - 225)} = 8$.
9. $\sqrt{(8^2 + 6^2)} = 10$.
10. distances 8 and 6 from centre $\rightarrow \text{gap} = 2$.

8.2 Angle at centre = $2 \times$ angle at circumference

1. Twice.
2. **40° .**
3. **70° .**
4. **70° .**
5. **100° .**
6. **60° .**
7. **50° .**
8. $2(2x) = 100 \rightarrow 4x = 100 \rightarrow x = 25^\circ$.
9. $x + 20 = 2(x - 10) \rightarrow x + 20 = 2x - 20 \rightarrow x = 40^\circ$.
10. Both stand on the same arc; the theorem follows from isosceles triangles formed by the radii.

8.2 Angle in semi-circle = 90°

1. **90° .**
2. **90° .**
3. $90 - 35 = 55^\circ$.
4. $90 - 40 = 50^\circ$.
5. A diameter.
6. **90° .**
7. $\angle C = 90^\circ \rightarrow BC = \sqrt{(100 - 36)} = 8$.
8. $\angle ACB = 90^\circ$, $\angle BAC = 65^\circ$.
9. **90°** (angle in a semicircle).
10. $\sqrt{(25 + 144)} = 13$.

8.2 Angles in same segment are equal

1. Equal.
2. **38° .**
3. **52° .**
4. $2x = 50 \rightarrow x = 25^\circ$.
5. $x + 10 = 2x - 20 \rightarrow x = 30^\circ$.
6. **40°** (same segment).
7. $3x = x + 30 \rightarrow x = 15 \rightarrow \angle P = 45^\circ$.
8. $2 \times 35^\circ = 70^\circ$.
9. $4x - 10 = 2x + 30 \rightarrow x = 20^\circ$.
10. Each is half the same central angle on the same arc.

8.2 Cyclic quadrilateral — opposite angles supplementary

1. Supplementary (add to 180°).
2. **100° .**
3. **85° .**
4. **$x = 70^\circ$.**
5. **75° .**
6. **92° .**





7. Opposite of $100^\circ \rightarrow 80^\circ$.
8. $3x + 30 = 180 \rightarrow x = 50^\circ$.
9. $4x + 20 = 180 \rightarrow x = 40 \rightarrow \angle B = 120^\circ$.
10. Both pairs then sum to 180° (angles total 360°), satisfying the converse $\rightarrow$ it is **cyclic**.

8.2 Tangent-chord theorem (alternate segment)

1. The angle between a tangent and a chord equals the angle in the alternate segment.
2. 50° .
3. 65° .
4. $2x = 70 \rightarrow x = 35^\circ$.
5. A diameter.
6. 48° .
7. $x + 10 = 2x - 30 \rightarrow x = 40^\circ$.
8. 55° .
9. $180 - 50 - 60 = 70^\circ$.
10. Both relate to the same arc cut off by the chord.

8.2 Two tangents from an external point

1. They are equal.
2. **9 cm**.
3. True.
4. PO bisects $\rightarrow 25^\circ$.
5. Isosceles $\rightarrow \angle APB = 180 - 2 \cdot 65 = 50^\circ$.
6. **7 m**.
7. $\sqrt{169 - 25} = 12$.
8. OAPB has right angles at A and B $\rightarrow \angle APB = 360 - 90 - 90 - 120 = 60^\circ$.
9. Each is $90^\circ \rightarrow 180^\circ$.
10. **Equilateral**.

End of Chapter 8 — Mixed Exercise

1. 50° .
2. Exactly one.
3. $\sqrt{100 - 36} = 8$, so the chord is 16.
4. 70° .
5. A diameter.
6. 40° (same segment).
7. Opposite of $100^\circ \rightarrow 80^\circ$.
8. 55° .
9. Each is $90^\circ \rightarrow 180^\circ$.
10. $90^\circ, 45^\circ, 45^\circ$.

CHAPTER 9 — Finance, growth and decay

9.1 Revision: compound interest

1. $A = 5000(1.08)^3 = \mathbf{R6,298.56}$.
2. $A = 10000(1.1)^2 = \mathbf{R12,100.00}$.
3. $A = \mathbf{R2,524.95}$.
4. $A = \mathbf{R12,308.99}$.
5. $A = \mathbf{R4,579.60}$; interest = **R579.60**.
6. $A = 12000(1.085)^6 = \mathbf{R19,577.61}$.
7. $i = 0.0075, n = 36 \rightarrow A = \mathbf{R19,629.68}$.
8. $i = 0.018, n = 20 \rightarrow A = \mathbf{R28,574.96}$.
9. $2 = 1.1^n \rightarrow n = \log 2 / \log 1.1 \approx \mathbf{7.27 \text{ years}}$.
10. $(1+i)^4 = 1.5 \rightarrow i = 1.5^{(1/4)} - 1 = \mathbf{10.6682\%}$.





9.2 Simple depreciation (straight line)

1. $A = 10000(1 - 0.10 \cdot 1) = \mathbf{R9,000.00}$.
2. $A = 10000(1 - 0.10 \cdot 3) = \mathbf{R7,000.00}$.
3. $A = 20000(1 - 0.30) = \mathbf{R14,000.00}$.
4. $A = 6000(1 - 0.80) = \mathbf{R1,200.00}$.
5. **12%** of the original each year.
6. $A = 250000(1 - 0.50) = \mathbf{R125,000.00}$.
7. $1 - 0.16n = 0 \rightarrow n = 1/0.16 = \mathbf{6.25 \text{ years}}$.
8. $8000(1 - 4i) = 3200 \rightarrow 1 - 4i = 0.4 \rightarrow 4i = 0.6 \rightarrow i = \mathbf{15\%}$.
9. $(18000 - 6000)/5 = \mathbf{R2\ 400 \text{ per year}}$.
10. $5000/40000 = \mathbf{12.5\%}$.

9.2 Compound depreciation (reducing balance)

1. $A = 10000(0.9) = \mathbf{R9,000.00}$.
2. $A = 10000(0.9)^3 = \mathbf{R7,290.00}$.
3. $A = 20000(0.85)^2 = \mathbf{R14,450.00}$.
4. $A = 6000(0.8)^3 = \mathbf{R3,072.00}$.
5. Reducing balance (it never reaches zero).
6. $A = 250000(0.82)^5 = \mathbf{R92,684.96}$.
7. $A = 35000(0.88)^4 = \mathbf{R20,989.34}$.
8. $(1 - i)^3 = 0.512 \rightarrow 1 - i = 0.8 \rightarrow i = \mathbf{20\%}$.
9. $0.84^n < 0.4 \rightarrow n > \log 0.4 / \log 0.84 \approx 5.26 \rightarrow n = \mathbf{6 \text{ years}}$.
10. $(1 - i)^4 = 0.5 \rightarrow i = 1 - 0.5^{(1/4)} = \mathbf{15.9104\%}$.

9.3 Timelines (multiple rates)

1. $A = 5000(1.08)^2(1.1)^3 = \mathbf{R7,762.39}$.
2. $\mathbf{R11,292.65}$.
3. $A = 10000(1.06)^3(1.07)^2 = \mathbf{R13,635.94}$.
4. $A = 2000(1.01)^{24} = \mathbf{R2,539.47}$.
5. Multiply by $(1 + i)$ for each compounding period.
6. $6000(1.09)^5 + 4000(1.09)^3 = \mathbf{R14,411.86}$.
7. $12000(1.1)^3(1.08)^2 = \mathbf{R18,629.74}$.
8. $(10000(1.09)^2 - 3000)(1.09)^3 = \mathbf{R11,501.15}$.
9. $i = 0.035, n = 8 \rightarrow A = \mathbf{R19,752.14}$.
10. $i = 0.02, n = 20 \rightarrow A = \mathbf{R29,718.95}$.

9.4 Nominal vs effective interest

1. $i = \mathbf{1\% \text{ per month}}$.
2. $(1.01)^{12} - 1 = \mathbf{12.6825\%}$.
3. $(1.025)^4 - 1 = \mathbf{10.3813\%}$.
4. $(1.04)^2 - 1 = \mathbf{8.16\%}$.
5. The effective rate.
6. $A \rightarrow \text{effective } 9.3807\% > 9.3\% \rightarrow \mathbf{\text{choose Bank A}}$.
7. $(1.0125)^{12} - 1 = \mathbf{16.0755\%}$.
8. $(1 + j/4)^4 = 1.0824 \rightarrow j = 4(1.0824^{(1/4)} - 1) \approx \mathbf{7.997\%}$.
9. $6\% \text{ monthly} \rightarrow \text{effective } 6.1678\% > 6.1\% \rightarrow \mathbf{6\% \text{ monthly}}$.
10. $(1.015)^{12} - 1 = \mathbf{19.5618\%}$.

End of Chapter 9 — Mixed Exercise

1. $A = 5000(1.08)^3 = \mathbf{R6,298.56}$.
2. $A = 10000(1 - 0.10 \cdot 3) = \mathbf{R7,000.00}$.
3. $A = 20000(0.85)^2 = \mathbf{R14,450.00}$.
4. $A = 2000(1.01)^{24} = \mathbf{R2,539.47}$.
5. The effective rate.
6. $A = 12000(1.085)^6 = \mathbf{R19,577.61}$.





7. $1 - 0.16n = 0 \rightarrow n = 1/0.16 = \mathbf{6.25 \text{ years.}}$
8. $(1 - i)^3 = 0.512 \rightarrow 1 - i = 0.8 \rightarrow \mathbf{i = 20\%}$.
9. $i = 0.035, n = 8 \rightarrow A = \mathbf{R19,752.14}$.
10. $(1.015)^{12} - 1 = \mathbf{19.5618\%}$.

CHAPTER 10 — Probability

10.1 Revision: complement

1. **0.7.**
2. **0.75.**
3. $5/6$.
4. **0.55.**
5. $7/10$.
6. **70%.**
7. **0.6.**
8. **0.35.**
9. $P(\text{fail}) = 0.2 \rightarrow 0.2 \times 200 = \mathbf{40 \text{ learners.}}$
10. $P(\text{working}) = 0.96 \rightarrow 0.96 \times 500 = \mathbf{480 \text{ items.}}$

10.2 Independent events

1. $P(A) \cdot P(B)$.
2. **0.2.**
3. $1/4$.
4. $1/36$.
5. **0.3.**
6. $0.7 \times 0.6 = \mathbf{0.42}$.
7. $0.4 \times 0.4 = \mathbf{0.16}$.
8. $\frac{1}{2} \times \frac{1}{6} = \mathbf{1/12}$.
9. $0.28 = 0.7 \cdot P(B) \rightarrow \mathbf{0.4}$.
10. $0.95^3 = \mathbf{0.8574}$.

10.2 Dependent events

1. $(2/3)(1/2) = \mathbf{1/3}$.
2. $(3/5)(2/4) = \mathbf{3/10}$.
3. Dependent.
4. $(4/5)(3/4) = \mathbf{3/5}$.
5. $(3/5)(2/4) = \mathbf{3/10}$.
6. $(12/20)(11/19) = \mathbf{33/95}$.
7. $(6/10)(5/9) = \mathbf{1/3}$.
8. $(4/52)(3/51) = \mathbf{1/221}$.
9. $P(BB) + P(WW) = (5/8)(4/7) + (3/8)(2/7) = 20/56 + 6/56 = \mathbf{26/56 = 13/28}$.
10. $1 - P(\text{no win}) = 1 - (8/10)(7/9) = 1 - 56/90 = \mathbf{34/90 = 17/45}$.

10.3 Venn diagrams

1. $A \cap B$ (both events occur).
2. $20 + 15 - 5 = \mathbf{30}$.
3. $30 + 25 - 10 = \mathbf{45}$.
4. $50 - 45 = \mathbf{5}$.
5. $18 - 7 = \mathbf{11}$.
6. $22 - 8 = \mathbf{14}$.
7. $40 - (22 + 18 - 8) = 40 - 32 = \mathbf{8}$.
8. $(60 + 45 - 25)/100 = \mathbf{0.8}$.
9. $(50 - 20) + (30 - 20) = \mathbf{40}$.
10. $(12 + 9)/30 = \mathbf{0.7}$.





10.4 Tree diagrams

1. 4.
2. $2^3 = 8$.
3. Multiplying.
4. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.
5. $0.6 \times 0.4 = 0.24$.
6. $(\frac{3}{5})(\frac{2}{4}) = \frac{3}{10}$.
7. $(\frac{1}{2})^3 = \frac{1}{8}$.
8. $0.4 \cdot 0.6 + 0.6 \cdot 0.4 = 0.48$.
9. $0.9 \times 0.9 = 0.81$.
10. $(\frac{4}{5})(\frac{1}{4}) + (\frac{1}{5})(\frac{4}{4}) = \frac{4}{20} + \frac{4}{20} = \frac{2}{5}$.

10.5 Contingency tables

1. The total number surveyed.
2. $30/80 = \frac{3}{8}$.
3. $40/50 = 0.8$.
4. $35/100 = 0.35$.
5. $P(A) \cdot P(B)$.
6. $(90 + 20)/200 = 0.55$.
7. $90/200 = 0.45$.
8. $0.6 \times 0.55 = 0.33 \neq 0.45 \rightarrow$ **not independent**.
9. $(70 + 10)/130 \dots$ use totals: vaccinated 100, unvaccinated 30 $\rightarrow (70 + 10)/130 = \frac{80}{130} \approx 0.62$.
10. $P(\text{boy and sport}) = 18/40$; among boys, $18/25 = 0.72$.

End of Chapter 10 — Mixed Exercise

1. 0.7.
2. 0.2.
3. Dependent.
4. $50 - 45 = 5$.
5. $0.6 \times 0.4 = 0.24$.
6. $(90 + 20)/200 = 0.55$.
7. 0.6.
8. $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$.
9. $P(BB) + P(WW) = (\frac{5}{8})(\frac{4}{7}) + (\frac{3}{8})(\frac{2}{7}) = \frac{20}{56} + \frac{6}{56} = \frac{26}{56} = \frac{13}{28}$.
10. $(12 + 9)/30 = 0.7$.

CHAPTER 11 — Statistics

11.1 Revision: 5-number summary

1. 8.
2. min 2, max 15.
3. $20 - 5 = 15$.
4. Lower half 2,4,6,8 $\rightarrow Q1 = 5$.
5. 8.
6. min 40, Q1 55, median 65, Q3 75, max 90.
7. $75 - 55 = 20$.
8. $(400 + 450)/2 = \text{R}425$.
9. min 10, Q1 13, median 17, Q3 21, max 24.
10. $IQR = 35 - 20 = 15$; range = $50 - 12 = 38$.





11.2 Histograms

1. The frequency (count) for that interval.
2. **20**.
3. Because the data is continuous (grouped intervals).
4. **[20-30)**.
5. The third class (frequency 12).
6. $3 + 8 = \mathbf{11 \text{ plants}}$.
7. **[5-10)**.
8. **30**.
9. $10 + 7 + 5 = \mathbf{22}$.
10. Mode interval **[40-60)**; the shape is roughly symmetric.

11.3 Ogives

1. Cumulative frequency against the upper boundary of each interval.
2. **25**.
3. 20 (the halfway value).
4. 5, 15, 23, 30.
5. The lower quartile, Q1.
6. Median is at the 30th value $\rightarrow$ about **55%**.
7. $20/80 = \mathbf{1/4}$.
8. $24 - 10 = \mathbf{14}$.
9. $62 - 30 = \mathbf{32}$.
10. $(50 - 40)/50 = \mathbf{20\%}$.

11.4 Variance

1. The mean of the squared deviations from the mean.
2. **4**.
3. mean 4 $\rightarrow ((-2)^2 + 0 + 2^2)/3 = \mathbf{2.67}$.
4. mean 4 $\rightarrow \mathbf{5}$.
5. **0** (no spread).
6. mean 70 $\rightarrow (100 + 0 + 100)/3 = \mathbf{66.67}$.
7. mean 5 $\rightarrow \mathbf{5}$.
8. $\{1, 5, 9\}$ (larger spread).
9. mean 14 $\rightarrow \mathbf{8}$.
10. It stays the same (spread is unchanged).

11.4 Standard deviation

1. It is the square root of the variance.
2. **4**.
3. **5**.
4. variance 2.67 $\rightarrow \sigma \approx \mathbf{1.63}$.
5. All values are identical (no spread).
6. $\sigma = \sqrt{(200/3)} \approx \mathbf{8.16}$.
7. Class A (smaller standard deviation).
8. variance 5 $\rightarrow \sigma \approx \mathbf{2.24}$.
9. **496 mL to 504 mL**.
10. It is multiplied by 3.

11.5 Symmetric vs skewed data

1. Symmetric.
2. Skewed right (positively).
3. Skewed left (negatively).
4. Positive (right) skew.
5. Symmetric.
6. Positively skewed (right).



7. Mean < median → **skewed left**.
8. Negative (left) skew.
9. mode < median < mean.
10. Positively skewed (right).

11.6 Identification of outliers

1. A value above $Q3 + 1.5 \cdot IQR$ is an outlier.
2. **10**.
3. $20 + 1.5 \cdot 10 = 35$.
4. $10 - 1.5 \cdot 10 = -5$.
5. Yes ($40 > 35$).
6. IQR 20; upper bound $70 + 30 = 100 \rightarrow 95 < 100 \rightarrow$ **no**.
7. IQR 200 → lower R0, upper **R800**.
8. Yes ($900 > 800$).
9. IQR 16; bounds -12 and $52 \rightarrow$ **only 55 is an outlier**.
10. It flags values unusually far from the middle 50%, not just the extremes.

End of Chapter 11 — Mixed Exercise

1. **8**.
2. **20**.
3. 20 (the halfway value).
4. mean 4 → **5**.
5. All values are identical (no spread).
6. Positively skewed (right).
7. IQR 200 → lower R0, upper **R800**.
8. $(400 + 450)/2 =$ **R425**.
9. $10 + 7 + 5 =$ **22**.
10. $(50 - 40)/50 =$ **20%**.

CHAPTER 12 — Linear programming

12.1 Linear programming: identify constraints

1. $x \leq 8$.
2. $y \geq 3$.
3. $x \geq 0, y \geq 0$.
4. $x + y \leq 10$.
5. $x \geq 2y$.
6. $x + y \leq 30$.
7. $2x + 3y \leq 60$.
8. $y \geq 5$.
9. $y \geq 3x$.
10. $6 \leq x \leq 14$.

12.1 Optimisation: corner points

1. A point where two boundary lines intersect.
2. **(2, 4)**.
3. **(4, 4)**.
4. **(0, 10)**.
5. **(8, 0)**.
6. **(0, 0), (6, 0), (0, 6)**.
7. Subtract: $x = 4, y = 6 \rightarrow$ **(4, 6)**.
8. $6 + 2y = 12 \rightarrow y = 3 \rightarrow$ **(2, 3)**.
9. **(0, 0), (5, 0), (5, 4), (0, 4)**.
10. $x + 2x + 1 = 7 \rightarrow x = 2, y = 5 \rightarrow$ **(2, 5)**.





12.1 Optimisation: evaluate objective at corners

1. $8 + 3 = 11$.
2. 7.
3. 12.
4. $10 + 12 = 22$.
5. All give 6 — they are equal.
6. $P = 900, 1100, 1200 \rightarrow \max$ **R1 200 at (30, 0)**.
7. $P = 0, 12, 10, 12 \rightarrow \max$ **12** (at (4,0) or (2,3)).
8. $C = 22, 16, 17 \rightarrow \min$ **16 at (3, 2)**.
9. $P = 150, 180, 160 \rightarrow \max$ **R180 at (10, 20)**.
10. A linear function increases steadily across the region, reaching its extreme on the boundary, at a vertex.

End of Chapter 12 — Mixed Exercise

1. $x \leq 8$.
2. (2, 4).
3. 12.
4. $x + y \leq 10$.
5. (8, 0).
6. $P = 900, 1100, 1200 \rightarrow \max$ **R1 200 at (30, 0)**.
7. $2x + 3y \leq 60$.
8. $6 + 2y = 12 \rightarrow y = 3 \rightarrow$ **(2, 3)**.
9. $P = 150, 180, 160 \rightarrow \max$ **R180 at (10, 20)**.
10. $6 \leq x \leq 14$.

CHAPTER 6 — Trigonometry

11.1 Trigonometric identities

1. 1.
2. $\sin\theta / \cos\theta$.
3. $\sin^2\theta$.
4. $\cos\theta$.
5. $\sin\theta \div (\sin\theta/\cos\theta) = \cos\theta$.
6. $\sin^2\theta/\sin\theta = \sin\theta$.
7. $(\sin\theta/\cos\theta) \cdot \cos\theta = \sin\theta$.
8. $-\sin^2\theta$.
9. $(1 - \cos^2\theta)/(1 - \cos\theta) = 1 + \cos\theta$.
10. $\cos\theta = 4/5, \tan\theta = 3/4$.

11.2 Reduction formula

1. $\sin\theta$.
2. $-\cos\theta$.
3. $-\sin\theta$.
4. $\cos\theta$.
5. $\tan\theta$.
6. $\cos\theta$.
7. $\cos(180^\circ - 30^\circ) = -\cos 30^\circ = -\sqrt{3}/2$.
8. $\sin(180^\circ + 30^\circ) = -\sin 30^\circ = -1/2$.
9. $(\sin\theta \cdot \cos\theta)/\sin\theta = \cos\theta$.
10. $\cos(360 - 60) + \sin(180 - 60) = \frac{1}{2} + \sqrt{3}/2 = (1 + \sqrt{3})/2$.



**11.3 Trigonometric equations**

1. $\theta = 30^\circ$ or 150° .
2. $\theta = 90^\circ$ or 270° .
3. $\theta = 45^\circ$ or 225° .
4. $\cos\theta = \sqrt{3}/2 \rightarrow \theta = 30^\circ$ or 330° .
5. $\theta = 210^\circ$ or 330° .
6. $\sin\theta = \frac{1}{2} \rightarrow \theta = 30^\circ$ or 150° .
7. $\theta = 60^\circ$ or 240° .
8. $\theta = 120^\circ$ or 240° .
9. $\sin\theta(2\sin\theta - 1) = 0 \rightarrow \theta = 0^\circ, 180^\circ, 360^\circ, 30^\circ, 150^\circ$.
10. $\theta = 30^\circ + 360^\circ k$ or $\theta = 150^\circ + 360^\circ k, k \in \mathbb{Z}$.

11.4 Area, sine, and cosine rules

1. $\frac{1}{2} \cdot 6 \cdot 8 \cdot \sin 30^\circ = 12$ sq units.
2. $b = 8 \sin 40^\circ / \sin 60^\circ \approx 5.94$.
3. $a^2 = 49 + 81 - 2 \cdot 7 \cdot 9 \cdot \cos 50^\circ \rightarrow a \approx 6.93$.
4. $a^2 = b^2 + c^2 - 2bc \cdot \cos A$.
5. $\frac{1}{2} \cdot 10 \cdot 12 \cdot \sin 45^\circ \approx 42.43$ sq units.
6. $\frac{1}{2} \cdot 50 \cdot 70 \cdot \sin 60^\circ \approx 1515.54$ m².
7. $\cos A = (49 + 64 - 25) / (2 \cdot 7 \cdot 8) \rightarrow A \approx 38.2^\circ$.
8. $d^2 = 64 + 25 - 2 \cdot 8 \cdot 5 \cdot \cos 70^\circ \rightarrow d \approx 7.85$ km.
9. $b = 10 \sin 75^\circ / \sin 40^\circ \approx 15.03$.
10. $\sqrt{(200^2 + 150^2 - 2 \cdot 200 \cdot 150 \cdot \cos 120^\circ)} \approx 304.14$ km.

End of Chapter 6 — Mixed Exercise

1. 1.
2. $-\cos\theta$.
3. $\theta = 45^\circ$ or 225° .
4. $a^2 = b^2 + c^2 - 2bc \cdot \cos A$.
5. $\sin\theta \div (\sin\theta/\cos\theta) = \cos\theta$.
6. $\cos\theta$.
7. $\theta = 60^\circ$ or 240° .
8. $d^2 = 64 + 25 - 2 \cdot 8 \cdot 5 \cdot \cos 70^\circ \rightarrow d \approx 7.85$ km.
9. $(1 - \cos^2\theta)/(1 - \cos\theta) = 1 + \cos\theta$.
10. $\cos(360 - 60) + \sin(180 - 60) = \frac{1}{2} + \sqrt{3}/2 = (1 + \sqrt{3})/2$.

CHAPTER 5 — Functions**11.F1 Straight Line function**

1. $m = 3$, y-intercept $(0, -2)$.
2. $(3, 0)$.
3. $y = 2x + 1$.
4. $x = -x + 4 \rightarrow (2, 2)$.
5. $m = -\frac{1}{2}$, y-intercept $(0, 3)$.
6. Fixed R12; rate R8 per km.
7. $8 \cdot 9 + 12 = \text{R}84$.
8. $20 - 2t = 0 \rightarrow t = 10$ hours.
9. $m = 3 \rightarrow y = 3x + 1$.
10. $2x + 1 = -x + 7 \rightarrow x = 2 \rightarrow (2, 5)$.



**11.F2 Quadratic function**

1. **(0, 3).**
2. $(x - 1)(x - 3) = 0 \rightarrow x = 1 \text{ and } 3.$
3. $x = -b/2a = x = 2.$
4. $x = 2, y = -1 \rightarrow (2, -1).$
5. A minimum ($a > 0$).
6. $-5t(t - 4) = 0 \rightarrow t = 4 \text{ s.}$
7. Axis $t = 2$; $h(2) = -20 + 40 = 20 \text{ m.}$
8. $x = -3, y = -4 \rightarrow (-3, -4).$
9. $x = -b/2a = 4 \text{ units.}$
10. $y = a(x - 1)(x - 5)$; $(0, 5)$: $5a = 5 \rightarrow a = 1 \rightarrow y = (x - 1)(x - 5).$

11.F3 Hyperbolic function

1. $x = 0$ and $y = 0.$
2. $x = 0$ and $y = 2.$
3. $x = 3$ and $y = 0.$
4. **3.**
5. **$x = 4.$**
6. $x = -1$ and $y = -2.$
7. $240/8 = 30 \text{ hours.}$
8. **24 workers.**
9. $x^2 = 4 \rightarrow (2, 2) \text{ and } (-2, -2).$
10. $a = 12 \rightarrow y = 12/x.$

11.F4 Exponential function

1. $y = 0.$
2. **8.**
3. $y = -4.$
4. $(0, 1).$
5. Decreasing ($0 < \text{base} < 1$).
6. **32.**
7. **R1,259.71.**
8. **R144,500.00.**
9. y-intercept $(0, 3)$; asymptote $y = 0.$
10. $50(0.5)^3 = 6.25 \text{ mg.}$

End of Chapter 5 — Mixed Exercise

1. $m = 3$, y-intercept $(0, -2).$
2. $(x - 1)(x - 3) = 0 \rightarrow x = 1 \text{ and } 3.$
3. $x = 3$ and $y = 0.$
4. $(0, 1).$
5. $m = -1/2$, y-intercept $(0, 3).$
6. $-5t(t - 4) = 0 \rightarrow t = 4 \text{ s.}$
7. $240/8 = 30 \text{ hours.}$
8. **R144,500.00.**
9. $m = 3 \rightarrow y = 3x + 1.$
10. $y = a(x - 1)(x - 5)$; $(0, 5)$: $5a = 5 \rightarrow a = 1 \rightarrow y = (x - 1)(x - 5).$



CHAPTER 1 — Exponents

11.E1 Rational exponents and surds

1. $5\sqrt{2}$.
2. $(\sqrt[3]{8})^2 = 4$.
3. $(\sqrt[4]{16})^3 = 8$.
4. $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$.
5. $x^{(1/2)}$.
6. $(\sqrt[3]{27})^2 = 9$.
7. $\sqrt{5/5}$.
8. $x(2 - \sqrt{3}) \rightarrow 2 - \sqrt{3}$.
9. $7 - 2 = 5$.
10. $(\sqrt[5]{32})^2 \cdot \sqrt{4} = 4 \cdot 2 = 8$.

11.E2 Solving surd equations

1. $x = 25$.
2. $x = 49$.
3. $x + 1 = 9 \rightarrow x = 8$.
4. $2x - 1 = 25 \rightarrow x = 13$.
5. $\sqrt{x} = 4 \rightarrow x = 16$.
6. $x + 4 = x^2 - 4x + 4 \rightarrow x^2 - 5x = 0 \rightarrow x = 0$ or 5 ; only $x = 5$ works.
7. It gives $2 = -2$, which is false.
8. $3x + 1 = x^2 - 2x + 1 \rightarrow x^2 - 5x = 0 \rightarrow x = 0$ or 5 ; check $\rightarrow x = 5$.
9. $x = 4^{(3/2)} = 8$.
10. $x - 3 = 2x - 7 \rightarrow x = 4$.

11.E3 Applications of exponentials (growth/decay)

1. $100 \cdot 8 = 800$.
2. **R6,050.00**.
3. **R92,160.00**.
4. **5 mg**.
5. $50 \cdot 9 = 450$.
6. **24333**.
7. $12000(0.75)^3 = \mathbf{R5,062.50}$.
8. $2^t = 16 \rightarrow t = \mathbf{4 \text{ hours}}$.
9. $1.09^t = 2 \rightarrow t = \log 2 / \log 1.09 \approx \mathbf{8.04 \text{ years}}$.
10. $(1/2)^3 = 1/8$.

End of Chapter 1 — Mixed Exercise

1. $5\sqrt{2}$.
2. $x = 49$.
3. **R92,160.00**.
4. $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$.
5. $\sqrt{x} = 4 \rightarrow x = 16$.
6. **24333**.
7. $\sqrt{5/5}$.
8. $3x + 1 = x^2 - 2x + 1 \rightarrow x^2 - 5x = 0 \rightarrow x = 0$ or 5 ; check $\rightarrow x = 5$.
9. $1.09^t = 2 \rightarrow t = \log 2 / \log 1.09 \approx \mathbf{8.04 \text{ years}}$.
10. $(\sqrt[5]{32})^2 \cdot \sqrt{4} = 4 \cdot 2 = 8$.





CHAPTER 3 — Number Patterns

11.P1 Arithmetic sequences

1. 19.
2. $d = 4$.
3. 29.
4. $T_n = 5n - 4$.
5. 820.
6. $a = 20, d = 4 \rightarrow T_{10} = 56$ seats.
7. R10500.
8. $a = 3, d = 4, n = 12 \rightarrow S = 300$ logs.
9. $4 + (n - 1)3 = 100 \rightarrow n = 33$.
10. $d = 4, a = 3 \rightarrow a = 3, d = 4$.

11.P2 Quadratic sequences

1. 2.
2. $2a = 2 \rightarrow a = 1$.
3. Diffs 5, 7, 9 $\rightarrow$ next 11 $\rightarrow 35$.
4. $T_n = n^2$.
5. $T_n = n^2 + n$.
6. 100 dots.
7. $T_n = n^2 + n$.
8. 2, 6, 13, 23 $\rightarrow T_4 = 23$.
9. Diffs 5, 9, 13 $\rightarrow$ next 17 $\rightarrow T_5 = 47$.
10. $(n - 2)(n - 3) = 0 \rightarrow n = 2$ or 3.

11.P3 Geometric sequences

1. 54.
2. $r = 2$.
3. 625.
4. 192.
5. $T_n = 2^{(n+1)}$.
6. $a = 16, r = \frac{1}{2} \rightarrow T_6 = 0.5$ m.
7. $3 \cdot 2^{(n-1)} = 96 \rightarrow 2^{(n-1)} = 32 \rightarrow n = 6$.
8. $T_4 = R102400$.
9. $r^2 = 9 \rightarrow r = 3$.
10. $(x + 3)^2 = x(x + 9) \rightarrow 9 = 3x \rightarrow x = 3$.

End of Chapter 3 — Mixed Exercise

1. 19.
2. $2a = 2 \rightarrow a = 1$.
3. 625.
4. $T_n = 5n - 4$.
5. $T_n = n^2 + n$.
6. $a = 16, r = \frac{1}{2} \rightarrow T_6 = 0.5$ m.
7. R10500.
8. 2, 6, 13, 23 $\rightarrow T_4 = 23$.
9. $r^2 = 9 \rightarrow r = 3$.
10. $d = 4, a = 3 \rightarrow a = 3, d = 4$.





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