

FIRE JUNGLE

CAPS-aligned mathematics for South African learners



GRADE 10

Mathematics

TEXTBOOK

Definitions • Worked Examples • Exercises • Full Answers



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by Joseph Matthew Mitchell

Fire Jungle Grade 10 Mathematics

A CAPS-aligned study textbook for the FET phase

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How to use this book

Definitions. Each topic opens with the key terms you must know and be able to state in an exam.

Key formulas. The formulas and rules for the topic, gathered in one box for quick revision.

Worked examples. Fully solved problems. The right-hand notes explain the reason for every step.

Exercises. Practice questions. Try them before checking the back of the book.

Answers. Complete answers with working are collected in the Answers section at the end.

Digital edition. Scan the QR code on any page to open the interactive version of this textbook, with animated lessons, narration and unlimited generated practice.

Tip: work through the worked examples with a pen in hand, then attempt the exercises with the book closed. Use the digital edition for extra randomised practice on any topic you find hard.



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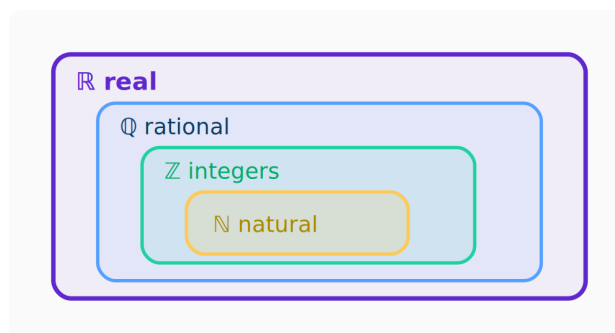


CHAPTER 1

Algebraic expressions

- 1.2 The real number system
- 1.3 Rational vs irrational numbers
- 1.4 Rounding off
- 1.5 Estimating surds
- 1.6 Products: distributive law
- 1.6 Products: binomial $\times$ binomial (FOIL)
- 1.6 Products: difference of squares
- 1.6 Products: perfect square trinomial
- 1.7 Factorisation: common factor
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- 1.7 Factorisation: trinomials $x^2 + bx + c$
- 1.7 Factorisation: trinomials with negative middle
- 1.7 Factorisation: grouping (4 terms)
- 1.8 Simplification of fractions

1.2 The real number system



DEFINITIONS



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Natural numbers ($\mathbb{N}$)	The counting numbers: 1, 2, 3, 4, ... (zero is excluded).
Whole numbers ($\mathbb{N}_0$)	Naturals together with zero: 0, 1, 2, 3, ...
Integers ($\mathbb{Z}$)	All whole numbers and their negatives: ..., -2, -1, 0, 1, 2, ...
Rational number ($\mathbb{Q}$)	Any number that can be written as p/q where p, q are integers and $q \neq 0$.
Irrational number ($\mathbb{Q}'$)	A real number that CANNOT be written as p/q . Its decimal never repeats and never terminates (e.g. π , $\sqrt{2}$).
Real number ($\mathbb{R}$)	Any rational or irrational number — every point on the number line is a real number.

KEY FORMULAS

- **Naturals ($\mathbb{N}$):** 1, 2, 3, 4, ...
- **Whole numbers ($\mathbb{N}_0$):** 0, 1, 2, 3, ...
- **Integers ($\mathbb{Z}$):** ..., -2, -1, 0, 1, 2, ...
- **Rationals ($\mathbb{Q}$):** any number that can be written as p/q where $q \neq 0$
- **Irrationals ($\mathbb{Q}'$):** cannot be written as p/q (e.g. π , $\sqrt{2}$, e)
- **Real numbers ($\mathbb{R}$):** rationals $\cup$ irrationals — every point on the number line

WORKED EXAMPLES

Example 1: Classify 7.

- 7 is a counting number, so it's **natural**.
Naturals start at 1.
- It's also whole, integer, rational, AND real.
Each category contains the previous.

Example 2: Classify $\sqrt{2}$.

- $\sqrt{2} \approx 1.41421356...$ — this decimal never repeats and never terminates.
Cannot be written as p/q .
- Therefore **irrational** (and real).
It IS a real number, just not rational.

Example 3: Classify -3.

- -3 is negative, so not natural and not whole.
Both of those start at 1 or 0.
- It IS an **integer**, and also rational and real.
Negative whole numbers are integers.

EXERCISES (EASY → DIFFICULT)

1. Is 7 a natural number?
2. Is 0 a whole number?
3. Is -5 an integer?
4. Classify $\frac{1}{2}$ (natural / integer / rational).
5. Is $\sqrt{2}$ rational or irrational?



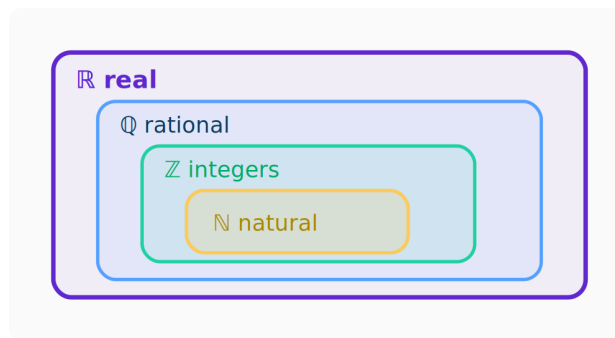
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6. To which sets does -3 belong: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$?
7. Give an example of a number that is real but not rational.
8. Is 0.75 rational? Write it as a fraction.
9. Is every integer a rational number?
10. Place 4 , -2 , $\frac{2}{3}$, $\sqrt{9}$ into the smallest set each belongs to.

1.3 Rational and irrational numbers

Rational vs irrational numbers



DEFINITIONS

Rational number	A number of the form p/q with integer p and q , $q \neq 0$.
Irrational number	A real number that is not rational; its decimal expansion is non-terminating and non-repeating.
Terminating decimal	A decimal that ends after a finite number of digits, e.g. 0.75 . Always rational.
Recurring decimal	A decimal with a digit pattern that repeats forever, e.g. $0.333\dots$. Always rational.

KEY FORMULAS

- **Rational:** p/q where p , q are integers and $q \neq 0$
- **Irrational:** cannot be written as p/q
- **Terminating decimal** $\Rightarrow$ rational (e.g. $0.5 = 1/2$)
- **Recurring decimal** $\Rightarrow$ rational (e.g. $0.333\dots = 1/3$)
- **Non-recurring, non-terminating** $\Rightarrow$ irrational (e.g. π , $\sqrt{2}$)
- **$\sqrt{n}$ is rational** only if n is a perfect square

WORKED EXAMPLES

Example 1: Is 0.625 rational?

- It terminates: $0.625 = 625/1000 = 5/8$.
Terminating decimals always give a fraction.
- **Yes — rational.**
Can be written as p/q with $q \neq 0$.



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**Example 2: Is $\sqrt{16}$ rational?**

► $\sqrt{16} = 4$, which is an integer.

16 is a perfect square.

► Integers are rational ($4 = 4/1$). **Yes — rational.**

Every integer is rational.

Example 3: Is π rational?

► $\pi = 3.14159265358979\dots$ — never repeats, never terminates.

Famously irrational.

► **No — irrational.**

Cannot be written as p/q .

EXERCISES (EASY → DIFFICULT)

1. Is 0.5 rational or irrational?
2. Is $\sqrt{16}$ rational?
3. Is π rational?
4. Write 0.333... as a fraction.
5. Is 0.101001000... (non-repeating) rational?
6. Between which two integers does $\sqrt{10}$ lie?
7. Is $\sqrt{7}$ rational? Give a reason.
8. Write 0.45 (recurring 45) as a fraction.
9. Classify $22/7$.
10. Show $0.999\dots$ equals 1.

1.4 Rounding off

rounding off**DEFINITIONS**

Rounding off	Replacing a number with one that has fewer digits, kept as close as possible to the original.
Decimal place	A digit position to the right of the decimal point. The first decimal place is tenths, the second is hundredths.
Significant figure	A digit that contributes to the precision of a number — leading zeros do not count.

KEY FORMULAS**DIGITAL TEXTBOOK**

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- **Step 1:** Identify the digit in the place you're rounding TO
- **Step 2:** Look at the digit immediately AFTER it
- **Step 3:** If that digit is 5 or more → round UP (add 1 to the previous digit)
- **Step 4:** If that digit is less than 5 → round DOWN (leave the previous digit)
- **Step 5:** Drop all digits to the right

WORKED EXAMPLES

Example 1: Round 3.14159 to 2 decimal places.

- ▶ Look at the 3rd decimal digit: **1**.

1 is less than 5.

- ▶ Round DOWN: **3.14**.

Drop the 159.

Example 2: Round 7.6543 to 1 decimal place.

- ▶ Look at the 2nd decimal digit: **5**.

5 means round up.

- ▶ Round UP: **7.7**.

6 becomes 7, drop the rest.

Example 3: Round 5.999 to 2 decimal places.

- ▶ Look at the 3rd decimal: **9**. Round up.

$9 \geq 5$.

- ▶ $5.99 + 0.01 = \mathbf{6.00}$.

Carrying ripples through 9s.

EXERCISES (EASY → DIFFICULT)

1. Round 3.467 to 2 decimal places.
2. Round 12.834 to 1 decimal place.
3. Round 0.0456 to 2 decimal places.
4. Round 149.5 to the nearest whole number.
5. Round 3.14159 to 3 decimal places.
6. A till total is R47.378. Round to the nearest cent.
7. A length is 2.8451 m. Round to 2 decimals.
8. Round 9 999.6 to the nearest thousand.
9. Petrol costs R23.456 per litre. Round to the nearest cent.
10. Round $0.6 \times R349$ to the nearest rand.



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1.5 Estimating surds

estimate the surd between integers



DEFINITIONS

Surd	A root expression like $\sqrt{2}$ or $\sqrt[3]{5}$ that cannot be simplified to a rational number.
Perfect square	A number that is the square of an integer: 1, 4, 9, 16, 25, 36, ...
Estimation	Finding a value close to the true answer by using sandwiching numbers.

KEY FORMULAS

- **Surd:** an irrational root like $\sqrt{2}$, $\sqrt{7}$, $\sqrt{20}$
- **Strategy:** sandwich the number between two perfect squares
- If $a^2 < n < b^2$ then $a < \sqrt{n} < b$
- **Useful perfect squares:** 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

WORKED EXAMPLES

Example 1: Between which integers does $\sqrt{7}$ lie?

- Squares around 7: $4 = 2^2 < 7 < 9 = 3^2$.

Find the bracket.

- Therefore $2 < \sqrt{7} < 3$.

Take square roots.

Example 2: Estimate $\sqrt{50}$.

- $49 = 7^2 < 50 < 64 = 8^2$.

50 is just above 49.

- $7 < \sqrt{50} < 8$; very close to 7 (≈ 7.07).

Since 50 is closer to 49.

Example 3: Estimate $\sqrt{90}$.

- $81 = 9^2 < 90 < 100 = 10^2$.

Locate between consecutive perfect squares.

- $9 < \sqrt{90} < 10$ (about 9.49).

Closer to 9 than 10.

EXERCISES (EASY → DIFFICULT)

1. Between which two integers does $\sqrt{20}$ lie?
2. Between which two integers does $\sqrt{50}$ lie?
3. Estimate $\sqrt{30}$ to the nearest integer.
4. Between which two integers does $\sqrt{8}$ lie?
5. Estimate $\sqrt{72}$ to the nearest integer.
6. Which is larger: $\sqrt{40}$ or 6?



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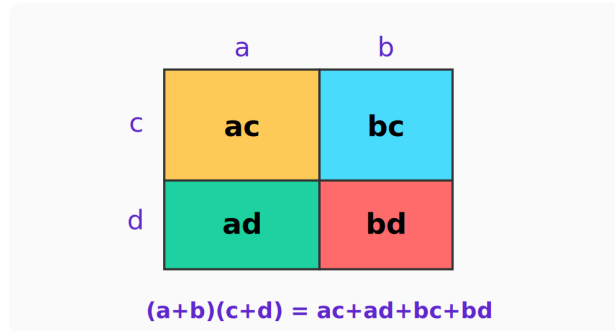
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7. Between which two integers does $\sqrt{128}$ lie?
8. A square garden has area 45 m^2 . Estimate its side to the nearest metre.
9. Order $\sqrt{5}$, 2, $\sqrt{8}$ from smallest.
10. A square tile has area 90 cm^2 . Between which two whole numbers (cm) is its side?

1.6 Products: distributive

Products: distributive law



DEFINITIONS

Distributive law

$a(b + c) = ab + ac$. The outside term multiplies EVERY term inside the bracket.

Like terms

Terms with exactly the same variable part — they can be combined by adding their coefficients.

KEY FORMULAS

- **Distributive law:** $a(b + c) = ab + ac$
- **Also:** $a(b - c) = ab - ac$
- **Three terms:** $a(b + c + d) = ab + ac + ad$
- **With variable outside:** $x(2x + 5) = 2x^2 + 5x$
- **Sign rule:** minus $\times$ minus = plus, minus $\times$ plus = minus

WORKED EXAMPLES

Example 1: Expand $3(x + 4)$.

- Multiply 3 by each term inside: $3 \cdot x + 3 \cdot 4$.

Distribute.

► $= 3x + 12$.

Done.

Example 2: Expand $x(2x - 5)$.

- $x \cdot 2x - x \cdot 5$.

Distribute the x .

► $= 2x^2 - 5x$.

$x \cdot x = x^2$.



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Example 3: Expand $-2(3a - 7)$.

$$\triangleright -2 \cdot 3a + (-2) \cdot (-7).$$

Watch the signs!

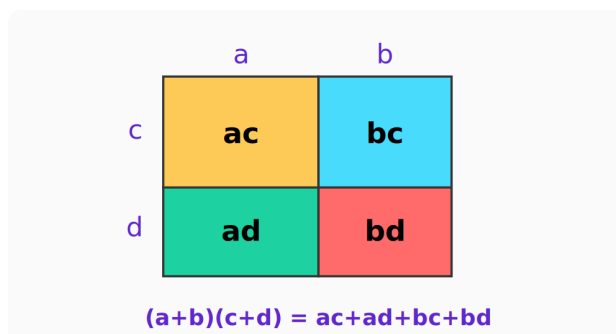
$$\triangleright = -6a + 14.$$

Negative times negative is positive.

EXERCISES (EASY → DIFFICULT)

1. Expand $3(x + 4)$.
2. Expand $5(2a - 3)$.
3. Expand $-2(x - 7)$.
4. Expand $x(x + 6)$.
5. Expand $4(2x^2 - x + 3)$.
6. Expand $2a(3a + 5)$.
7. Expand $-3x(x - 2y)$.
8. Expand and simplify $2(x + 3) + 3(x - 1)$.
9. Expand $5(x - 2) - 2(x + 4)$.
10. A rectangle has width 4 and length $(x + 3)$. Write its area expanded.

1.6 Products: binomial $\times$ binomial (FOIL)



DEFINITIONS

Binomial An algebraic expression with exactly two terms, e.g. $(x + 3)$ or $(2a - 5)$.

FOIL A memory aid for multiplying two binomials: First, Outer, Inner, Last.

KEY FORMULAS

- **FOIL:** First, Outer, Inner, Last
- $(a + b)(c + d) = ac + ad + bc + bd$
- **First:** $a \cdot c$ **Outer:** $a \cdot d$
- **Inner:** $b \cdot c$ **Last:** $b \cdot d$
- **Like terms** (those with the same variable powers) must be combined at the end

WORKED EXAMPLES



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**Example 1: Expand $(x + 3)(x + 5)$.**

► **F:** $x \cdot x = x^2$. **O:** $x \cdot 5 = 5x$. **I:** $3 \cdot x = 3x$. **L:** $3 \cdot 5 = 15$.

Apply FOIL.

► **Combine:** $x^2 + 5x + 3x + 15 = x^2 + 8x + 15$.

Add like terms $5x + 3x$.

Example 2: Expand $(2x - 1)(x + 4)$.

► **F:** $2x \cdot x = 2x^2$. **O:** $2x \cdot 4 = 8x$. **I:** $-1 \cdot x = -x$. **L:** $-1 \cdot 4 = -4$.

Watch the negative.

► **=** $2x^2 + 8x - x - 4 = 2x^2 + 7x - 4$.

Combine $8x - x = 7x$.

Example 3: Expand $(a - 2)(a - 3)$.

► **F:** a^2 . **O:** $-3a$. **I:** $-2a$. **L:** 6.

Both negative $\times$ negative = positive 6.

► **=** $a^2 - 5a + 6$.

Combine middle terms.

EXERCISES (EASY → DIFFICULT)

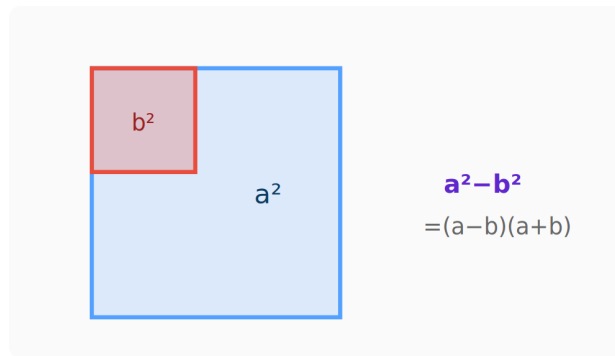
1. Expand $(x + 2)(x + 5)$.
2. Expand $(x + 3)(x - 4)$.
3. Expand $(x - 2)(x - 6)$.
4. Expand $(2x + 1)(x + 3)$.
5. Expand $(3x - 2)(x + 4)$.
6. Expand $(x + 5)(2x - 1)$.
7. Expand $(2x - 3)(3x - 1)$.
8. A rectangle is $(x + 4)$ by $(x + 1)$. Write its area expanded.
9. Expand $(x + 2)(x - 2)$ using FOIL.
10. Expand and simplify $(x + 3)(x + 2) - x^2$.

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1.6 Products: difference of squares



DEFINITIONS

Difference of squares An expression of the form $a^2 - b^2$. It always factorises as $(a + b)(a - b)$.

KEY FORMULAS

- $(a + b)(a - b) = a^2 - b^2$
- Spotting the pattern: same two terms, one with +, one with -
- The **middle terms cancel** exactly
- Reverse direction (factorising): $a^2 - b^2 = (a + b)(a - b)$

WORKED EXAMPLES

Example 1: Expand $(x + 4)(x - 4)$.

- Identify $a = x$, $b = 4$. Pattern matches.
Same first term, opposite signs on second.
- $= x^2 - 4^2 = x^2 - 16$.
Apply the formula directly.

Example 2: Expand $(2x + 5)(2x - 5)$.

- $a = 2x$, $b = 5$.
Watch the coefficient.
- $= (2x)^2 - 5^2 = 4x^2 - 25$.
 $(2x)^2 = 4x^2$.

Example 3: Expand $(3y - 7)(3y + 7)$.

- Order doesn't matter — still $a = 3y$, $b = 7$.
Pattern is symmetric.
- $= (3y)^2 - 49 = 9y^2 - 49$.
 $(3y)^2 = 9y^2$.

EXERCISES (EASY → DIFFICULT)

1. Expand $(x + 3)(x - 3)$.
2. Expand $(x - 5)(x + 5)$.
3. Expand $(2x + 1)(2x - 1)$.
4. Expand $(3x - 4)(3x + 4)$.
5. Expand $(a + 7)(a - 7)$.

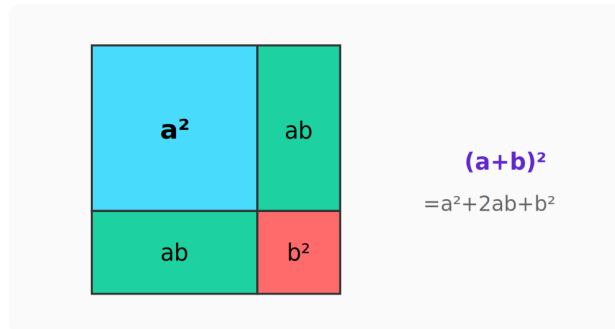


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6. Use the difference of squares to compute 101×99 .
7. Expand $(5 - x)(5 + x)$.
8. Compute 53×47 via difference of squares.
9. Expand $(xy + 2)(xy - 2)$.
10. Expand $(2a + 3b)(2a - 3b)$.

1.6 Products: perfect square trinomial



DEFINITIONS

Perfect square trinomial A trinomial that is the square of a binomial: $a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$.

KEY FORMULAS

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- **Common mistake:** $(a + b)^2 \neq a^2 + b^2$!
- Spotting: middle term is exactly $2 \times (\text{square root of first}) \times (\text{square root of last})$

WORKED EXAMPLES

Example 1: Expand $(x + 3)^2$.

► $a = x, b = 3$.

Identify the parts.

► $= x^2 + 2(x)(3) + 3^2 = x^2 + 6x + 9$.

Apply the formula.

Example 2: Expand $(2x - 5)^2$.

► $a = 2x, b = 5$; the minus needs care.

Use the $(a - b)^2$ version.

► $= (2x)^2 - 2(2x)(5) + 5^2 = 4x^2 - 20x + 25$.

Middle term is negative.

Example 3: Common mistake — what does $(x + 4)^2$ NOT equal?

► **WRONG:** $x^2 + 16$. (Forgetting the cross-product $2 \cdot x \cdot 4 = 8x$.)

Squares of sums are NOT sums of squares.

► **CORRECT:** $x^2 + 8x + 16$.

Always three terms with the cross $2ab$ in the middle.



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**EXERCISES (EASY → DIFFICULT)**

1. Expand $(x + 3)^2$.
2. Expand $(x - 4)^2$.
3. Expand $(2x + 1)^2$.
4. Expand $(3x - 2)^2$.
5. Expand $(x + 5)^2$.
6. Compute 21^2 using $(20 + 1)^2$.
7. Expand $(a - b)^2$.
8. Compute 99^2 using $(100 - 1)^2$.
9. Expand $(2x + 3y)^2$.
10. A square has side $(x + 6)$. Write its area expanded.

1.7 Factorisation: common factor**common factor**

take out / split into brackets

DEFINITIONS

Common factor A factor that appears in every term of an expression.

Highest common factor (HCF) The largest factor shared by every term — pull this one outside the bracket first.

KEY FORMULAS

- **HCF method:** $ax + ay = a(x + y)$
- **Find the largest factor** dividing every term
- **Apply to numbers AND variables:** lowest power of each shared variable
- **Always check** by expanding back

WORKED EXAMPLES**Example 1: Factorise $6x + 9$.**

► HCF of 6 and 9 is **3**. Pull it out: $3(\dots + \dots)$.

Take 3 from each.

► = **$3(2x + 3)$** .

Check: $3 \cdot 2x + 3 \cdot 3 = 6x + 9$

Example 2: Factorise $4x^2 - 6x$.

► HCF of 4 and 6 is 2. HCF of x^2 and x is x . Combined HCF = **$2x$** .

Take the lowest power of x .

► = **$2x(2x - 3)$** .

Check: $2x \cdot 2x - 2x \cdot 3 = 4x^2 - 6x$

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Example 3: Factorise $12a^2b + 18ab^2$.

► Numbers: $\text{HCF}(12, 18) = 6$. Variables: each term has at least 1 a and 1 b.

Take lowest power of each.

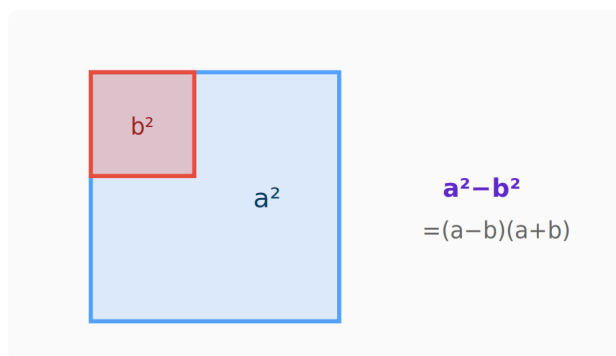
► $\text{HCF} = 6ab$. Result: **$6ab(2a + 3b)$** .

Check: $6ab \cdot 2a + 6ab \cdot 3b = 12a^2b + 18ab^2$

EXERCISES (EASY → DIFFICULT)

- Factorise $6x + 9$.
- Factorise $4a - 8$.
- Factorise $x^2 + 5x$.
- Factorise $12y^2 - 8y$.
- Factorise $15a^2b + 10ab^2$.
- Factorise $3x^2 + 6x + 9$.
- Factorise $-2x^2 - 4x$.
- Factorise $7x^3 - 14x^2 + 21x$.
- Factorise $a(x + 1) + b(x + 1)$.
- Factorise $2\pi r + 2\pi R$ (common factor).

1.7 Factorisation: difference of squares



DEFINITIONS

Difference of squares $a^2 - b^2$ factorises as $(a - b)(a + b)$.

Sum of squares $a^2 + b^2$ does NOT factor over the real numbers.

KEY FORMULAS

- **$a^2 - b^2 = (a + b)(a - b)$**
- **Spot it:** two perfect-square terms separated by a MINUS
- **NOT $a^2 + b^2$** — sum of squares does not factor over reals
- Both terms must be perfect squares (numbers and variables)



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WORKED EXAMPLES

Example 1: Factorise $x^2 - 9$.

► x^2 is a square; $9 = 3^2$. Minus sign — pattern matches.

Both squares, separated by minus.

► = **$(x + 3)(x - 3)$** .

Apply the formula directly.

Example 2: Factorise $4x^2 - 25$.

► $4x^2 = (2x)^2$. $25 = 5^2$. Pattern matches.

Identify the squares.

► = **$(2x + 5)(2x - 5)$** .

Apply: $a = 2x$, $b = 5$.

Example 3: Factorise $x^2 + 16$.

► Both squares, but the sign is PLUS — sum of squares.

Watch out!

► **Cannot be factorised** over real numbers.

Only difference (minus) works.

EXERCISES (EASY → DIFFICULT)

1. Factorise $x^2 - 16$.
2. Factorise $x^2 - 49$.
3. Factorise $4x^2 - 9$.
4. Factorise $25 - x^2$.
5. Factorise $9x^2 - 16$.
6. Factorise $a^2 - b^2$.
7. Factorise $2x^2 - 18$.
8. Factorise $x^4 - 1$.
9. Factorise $49a^2 - 81b^2$.
10. Factorise $x^2 - \frac{1}{4}$.



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1.7 Factorisation: trinomials $x^2 + bx + c$

$$x^2 + bx + c \rightarrow (x + _)(x + _)$$

take out / split into brackets

DEFINITIONS

Trinomial	An expression with three terms, e.g. $x^2 + 5x + 6$.
Quadratic trinomial (monic)	$x^2 + bx + c$ — find two numbers that MULTIPLY to c and ADD to b .

KEY FORMULAS

- **Goal:** factorise $x^2 + bx + c$ into $(x + p)(x + q)$
- **Need two numbers p, q such that:**
 - $p + q = b$ (sum gives middle coefficient)
 - $p \times q = c$ (product gives constant)
- **Both positive** if b and c are both positive

WORKED EXAMPLES

Example 1: Factorise $x^2 + 5x + 6$.

- Need p, q with $pq = 6$ and $p+q = 5$.

Set up the conditions.

- Factor pairs of 6: $1 \times 6, 2 \times 3$. Sums: 7, 5. **2 and 3 work.**

Check sums.

- = **$(x + 2)(x + 3)$** .

Verify by FOIL: $x^2 + 3x + 2x + 6 = x^2 + 5x + 6$

Example 2: Factorise $x^2 + 7x + 12$.

- $pq = 12, p+q = 7$. Pairs: $1 \cdot 12$ (sum 13), $2 \cdot 6$ (8), $3 \cdot 4$ (7)

Find the right pair.

- = **$(x + 3)(x + 4)$** .

Use 3 and 4.

Example 3: Factorise $x^2 + 8x + 15$.

- $pq = 15, p+q = 8$. Pairs: $1 \cdot 15$ (16), $3 \cdot 5$ (8)

3 and 5 work.

- = **$(x + 3)(x + 5)$** .

Done.

EXERCISES (EASY → DIFFICULT)

1. Factorise $x^2 + 5x + 6$.
2. Factorise $x^2 + 7x + 12$.
3. Factorise $x^2 + 8x + 15$.
4. Factorise $x^2 + 6x + 8$.



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5. Factorise $x^2 + 9x + 20$.
6. Factorise $x^2 + 2x + 1$.
7. Factorise $x^2 + 10x + 21$.
8. Factorise $x^2 + 11x + 24$.
9. Factorise $x^2 + 13x + 40$.
10. A rectangle has area $x^2 + 7x + 10$. Give possible side lengths.

1.7 Factorisation: trinomials with negative middle

split the middle term

take out / split into brackets

DEFINITIONS

Sign rule for trinomials

If c is positive, the two factors share the sign of b . If c is negative, the factors have OPPOSITE signs.

KEY FORMULAS

- $x^2 - bx + c$ ($b, c > 0$): both factors NEGATIVE
- $x^2 + bx - c$ or $x^2 - bx - c$: factors have OPPOSITE signs
- **Sign of c :** $c > 0 \rightarrow$ same signs; $c < 0 \rightarrow$ opposite signs
- **Sign of b :** tells which one dominates

WORKED EXAMPLES

Example 1: Factorise $x^2 - 7x + 10$.

- $c = 10 > 0 \rightarrow$ same signs. $b = -7 < 0 \rightarrow$ both NEGATIVE.

Read the sign rules.

- Need $pq = 10$ and $p+q = -7$. Pairs: $-2 \cdot -5$ (sum -7)

Find negatives.

- = **$(x - 2)(x - 5)$** .

Both negative.

Example 2: Factorise $x^2 + 2x - 8$.

- $c = -8 < 0 \rightarrow$ opposite signs.

Different signs.

- Need $pq = -8$ and $p+q = +2$. Try $+4 \cdot -2$: product -8 sum $+2$
 $+4$ dominates.

- = **$(x + 4)(x - 2)$** .

Right pair.



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**Example 3: Factorise $x^2 - x - 6$.**

► $c = -6 \rightarrow$ opposite signs. $b = -1 \rightarrow$ negative one dominates.

Read the signs.

► Try $+2 \cdot -3$: $pq = -6$, $\text{sum} = -1$

Verify.

► $= (x + 2)(x - 3)$.

Done.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. Factorise $x^2 - 5x + 6$.
2. Factorise $x^2 - 7x + 12$.
3. Factorise $x^2 - x - 6$.
4. Factorise $x^2 + 2x - 8$.
5. Factorise $x^2 - 3x - 10$.
6. Factorise $x^2 - 9x + 20$.
7. Factorise $x^2 + x - 12$.
8. Factorise $x^2 - 4x - 21$.
9. Factorise $x^2 - 2x - 15$.
10. Factorise $x^2 - 6x + 8$.

1.7 Factorisation: grouping (4 terms)

group in pairs

take out / split into brackets

DEFINITIONS**Factorisation
by grouping**

A method for factorising 4 terms: pair them, factor each pair, then take out the common bracket.

KEY FORMULAS

- **Use grouping when you have FOUR terms** with no obvious common factor across all
- **Step 1:** group into pairs: $(\text{term1} + \text{term2}) + (\text{term3} + \text{term4})$
- **Step 2:** factor a common factor from each pair
- **Step 3:** the two resulting brackets should be IDENTICAL — pull THAT out
- **Result:** $(\text{common bracket})(\text{other factors})$

WORKED EXAMPLES**DIGITAL TEXTBOOK**

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**Example 1: Factorise $xy + 3x + 2y + 6$.**

- Group: $(xy + 3x) + (2y + 6)$.

Pair adjacent terms.

- Factor each: $x(y + 3) + 2(y + 3)$.

Common factor in each pair.

- Both have $(y + 3)$. Pull it out: $(y + 3)(x + 2)$.

Final factorisation.

Example 2: Factorise $2x^2 + 6x + 5x + 15$.

- Group: $(2x^2 + 6x) + (5x + 15)$.

Pair up.

- Factor: $2x(x + 3) + 5(x + 3)$.

Same bracket appears!

- = $(x + 3)(2x + 5)$.

Pull out $(x + 3)$.

Example 3: Factorise $ab - 4a + 3b - 12$.

- Group: $(ab - 4a) + (3b - 12)$.

Pair up.

- Factor: $a(b - 4) + 3(b - 4)$.

Same bracket again.

- = $(b - 4)(a + 3)$.

Done.

EXERCISES (EASY → DIFFICULT)

1. Factorise by grouping: $x^3 + x^2 + 2x + 2$.
2. Factorise $ax + ay + bx + by$.
3. Factorise $2x + 2y + ax + ay$.
4. Factorise $x^3 - 3x^2 + 2x - 6$.
5. Factorise $3x + xy + 3 + y$.
6. Factorise $ab - 2a + 3b - 6$.
7. Factorise $x^2 + xy + 5x + 5y$.
8. Factorise $6 + 2x + 3y + xy$.
9. Factorise $2ax - 4a + 3x - 6$.
10. Factorise $pq - p - q + 1$.

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1.8 Simplification of fractions

factorise → cancel common factors

$$\frac{\text{numerator}}{\text{denominator}}$$

DEFINITIONS

Algebraic fraction

A fraction whose numerator and/or denominator contain variables, e.g. $(x^2 - 4)/(x - 2)$.

Cancel only factors

You may cancel only factors that are MULTIPLIED through. Never cancel terms that are added or subtracted.

KEY FORMULAS

- **Simplify:** factorise top and bottom, then cancel common factors
- **Cancel only FACTORS**, never terms
- **Look for:** common factors, difference of squares, trinomials
- **Restriction:** the cancelled factor cannot be zero (state this if asked)

WORKED EXAMPLES

Example 1: Simplify $(2x + 6)/(x^2 - 9)$.

► Top: $2(x + 3)$. Bottom: $(x + 3)(x - 3)$.

Factorise both.

► Cancel $(x + 3)$: $2/(x - 3)$, $x \neq \pm 3$.

State restriction.

Example 2: Simplify $(x^2 + 5x + 6)/(x + 2)$.

► Top: factor as $(x + 2)(x + 3)$. Bottom is already $(x + 2)$.

Trinomial factorisation.

► Cancel: $x + 2$, $x \neq -2$.

Restriction from cancelled factor.

Example 3: Simplify $(x^2 - 4)/(x^2 - 4x + 4)$.

► Top: $(x + 2)(x - 2)$. Bottom: $(x - 2)^2$.

Different patterns!

► Cancel ONE $(x - 2)$: $(x + 2)/(x - 2)$, $x \neq 2$.

Only one factor of $(x - 2)$ cancels.

EXERCISES (EASY → DIFFICULT)

1. Simplify $(2x)/(4)$.
2. Simplify $(6x^2)/(3x)$.
3. Simplify $(x^2 - 9)/(x - 3)$.
4. Simplify $(x^2 + 5x + 6)/(x + 2)$.
5. Simplify $(4x + 8)/(x + 2)$.
6. Simplify $(x^2 - 4)/(x^2 - 4x + 4)$.



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7. Simplify $(2/x) + (3/x)$.
8. Simplify $(1/2) + (1/3)$.
9. Simplify $(3x^2y)/(9xy^2)$.
10. Simplify $(x^2 - x - 6)/(x^2 - 9)$.

End of Chapter 1 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Is 7 a natural number?
2. Is $\sqrt{16}$ rational?
3. Round 0.0456 to 2 decimal places.
4. Between which two integers does $\sqrt{8}$ lie?
5. Expand $4(2x^2 - x + 3)$.
6. Expand $(x + 5)(2x - 1)$.
7. Expand $(5 - x)(5 + x)$.
8. Compute 99^2 using $(100 - 1)^2$.
9. Factorise $a(x + 1) + b(x + 1)$.
10. Factorise $x^2 - \frac{1}{4}$.



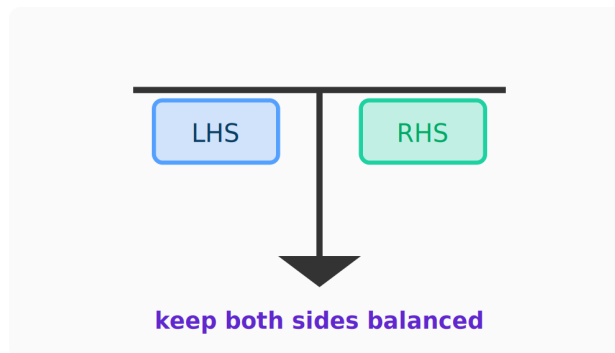
CHAPTER 4

Equations and inequalities

- 4.2 Linear equations: one-step
- 4.2 Linear equations: two-step
- 4.2 Linear equations with brackets
- 4.2 Variable on both sides
- 4.3 Quadratic equations: factorising
- 4.3 Quadratic: difference of squares
- 4.4 Simultaneous equations
- 4.5 Word problems with linear equations
- 4.6 Literal equations (changing the subject)
- 4.7 Linear inequalities
- 4.7 Inequality with negative coefficient

4.2 Solving linear equations (one-step)

Linear equations: one-step



DEFINITIONS

Linear equation	An equation where the unknown appears only to the first power, e.g. $x + 3 = 7$.
Solution / root	A value of the variable that makes the equation true.
Inverse operation	The operation that "undoes" another: addition undoes subtraction, multiplication undoes division.



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KEY FORMULAS

- **Add the same number to BOTH sides:** if $x - a = b$, then $x = a + b$
- **Subtract the same number from BOTH sides:** if $x + a = b$, then $x = b - a$
- **Multiply BOTH sides:** if $x \div a = b$, then $x = a \cdot b$
- **Divide BOTH sides:** if $a \cdot x = b$, then $x = b \div a$

WORKED EXAMPLES

Example 1: Solve $x + 7 = 15$.

$$\triangleright x + 7 - 7 = 15 - 7$$

Subtract 7 from both sides (inverse of +7).

$$\triangleright x = 8$$

Check: $8 + 7 = 15$

Example 2: Solve $4x = 20$.

$$\triangleright 4x \div 4 = 20 \div 4$$

Divide both sides by 4 (inverse of $\times 4$).

$$\triangleright x = 5$$

Check: $4 \times 5 = 20$

Example 3: Solve $x - 9 = -2$.

$$\triangleright x - 9 + 9 = -2 + 9$$

Add 9 to both sides.

$$\triangleright x = 7$$

Check: $7 - 9 = -2$

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. Solve $x + 5 = 12$.
2. Solve $x - 3 = 9$.
3. Solve $4x = 20$.
4. Solve $x/3 = 6$.
5. Solve $x + 7 = 2$.
6. Solve $5x = 35$.
7. Lerato had some sweets, ate 4, and has 11 left. How many did she start with?
8. A number tripled is 27. Find it.
9. Solve $-2x = 14$.
10. After spending R8 a learner has R12. How much did she have?

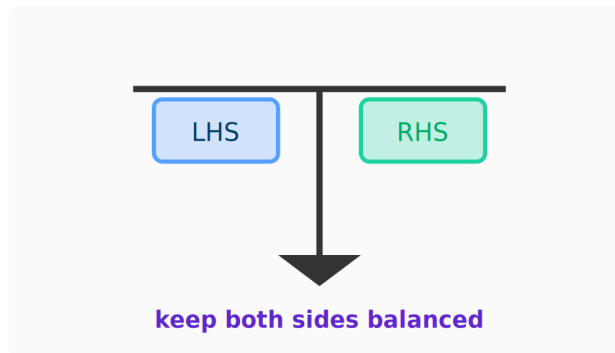


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4.2 Solving linear equations (two-step)

Linear equations: two-step



DEFINITIONS

Two-step equation

A linear equation that needs TWO inverse operations to isolate x , e.g. $2x + 3 = 11$.

Order of undoing

Undo addition/subtraction FIRST, then multiplication/division. (Reverse of BODMAS.)

KEY FORMULAS

- **Form:** $ax + b = c$
- **Step 1:** $ax = c - b$ (move b across)
- **Step 2:** $x = (c - b) \div a$ (divide by the coefficient)

WORKED EXAMPLES

Example 1: Solve $3x + 4 = 19$.

► $3x = 19 - 4 = 15$

Subtract 4 from both sides.

► $x = 15 \div 3$

Divide by the coefficient 3.

► $x = 5$

Check: $3(5) + 4 = 19$

Example 2: Solve $5x - 8 = 12$.

► $5x = 12 + 8 = 20$

Add 8 to both sides.

► $x = 4$

Check: $5(4) - 8 = 12$

Example 3: Solve $2x + 9 = 1$.

► $2x = 1 - 9 = -8$

Subtract 9.

► $x = -4$

Negative answers are fine.



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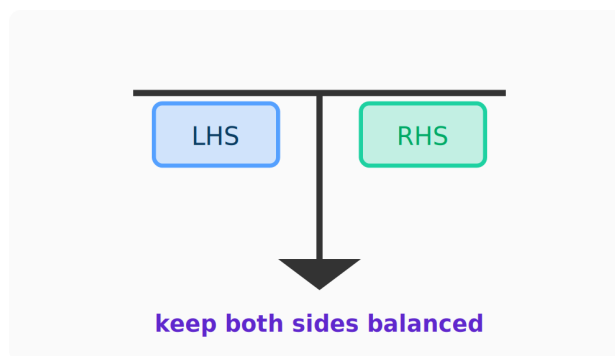
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**EXERCISES (EASY → DIFFICULT)**

1. Solve $2x + 3 = 11$.
2. Solve $3x - 5 = 16$.
3. Solve $5x + 2 = 17$.
4. Solve $4x - 7 = 9$.
5. Solve $(x/2) + 1 = 5$.
6. Solve $7 - 2x = 1$.
7. A taxi charges R5 plus R3 per km. A trip cost R20. How far?
8. Solve $6x + 4 = 28$.
9. A phone plan is R50 plus R2 per call; a bill is R80. How many calls?
10. Solve $(2x - 1)/3 = 5$.

4.2 Linear with brackets

Linear equations with brackets

**DEFINITIONS****Distributive expansion**

$a(b + c) = ab + ac$ — multiply the term outside through every term inside.

KEY FORMULAS

- **Step 1:** Expand brackets using $a(b + c) = ab + ac$
- **Step 2:** Collect like terms
- **Step 3:** Solve as a normal linear equation

WORKED EXAMPLES**Example 1: Solve $2(x + 3) = 14$.**

► $2x + 6 = 14$

Expand: $2 \times x + 2 \times 3$.

► $2x = 8$

Subtract 6.

► $x = 4$

Divide by 2.

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Example 2: Solve $5(x - 2) = 3x + 4$.

► $5x - 10 = 3x + 4$

Expand left side.

► $5x - 3x = 4 + 10$

Collect x's left, constants right.

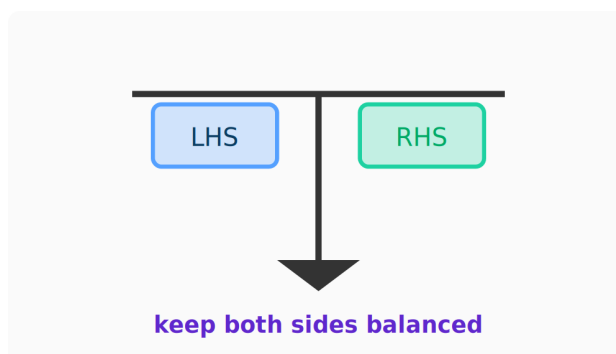
► $2x = 14$, so $x = 7$

Divide by 2.

EXERCISES (EASY → DIFFICULT)

1. Solve $2(x + 3) = 14$.
2. Solve $3(x - 2) = 12$.
3. Solve $4(2x - 1) = 20$.
4. Solve $5(x + 1) = 3(x + 5)$.
5. Solve $2(x - 3) + 4 = 10$.
6. Solve $3(2x + 1) - 2(x - 1) = 17$.
7. Solve $-(x - 4) = 2$.
8. Solve $2(3x - 1) = 4(x + 2)$.
9. A rectangle's perimeter is $2(x + 3) + 2x = 26$. Find x .
10. Solve $4(x + 2) - 3(x - 1) = 15$.

4.2 Variable on both sides


DEFINITIONS

Variable terms Terms containing the unknown (e.g. $5x$, $-2x$). Move them to ONE side of the equation.

Constant terms Terms without a variable (e.g. 7 , -3). Move them to the OTHER side.

KEY FORMULAS

- **Goal:** all x -terms on one side, constants on the other
- **Then:** divide by the resulting coefficient of x

WORKED EXAMPLES

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**Example 1: Solve $4x + 1 = x + 13$.**

► $4x - x = 13 - 1$

Subtract x from both sides AND subtract 1.

► $3x = 12$, $x = 4$

Divide by 3.

Example 2: Solve $7x - 5 = 3x + 11$.

► $7x - 3x = 11 + 5$

Group x -terms left, constants right.

► $4x = 16$, $x = 4$

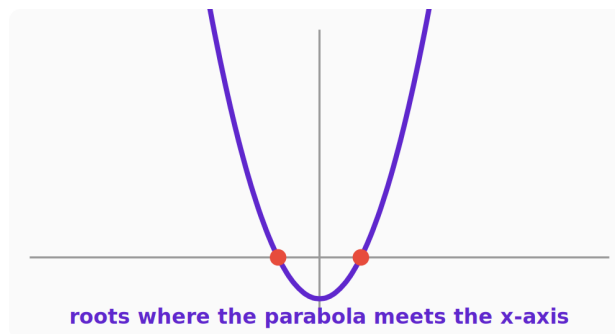
Divide by 4.

EXERCISES (EASY → DIFFICULT)

1. Solve $5x = 2x + 9$.
2. Solve $4x - 3 = 2x + 7$.
3. Solve $6x + 1 = 3x + 13$.
4. Solve $7x - 2 = 4x + 10$.
5. Solve $2x + 5 = 5x - 4$.
6. Solve $8 - x = 3x$.
7. Solve $3(x + 1) = x + 9$.
8. Two siblings: Sipho has x sweets, Thabo has $2x - 6$, and they are equal. Find x .
9. Solve $5x - 8 = 2x + 1$.
10. Solve $10 - 2x = x + 1$.

4.3 Solve quadratic by factorising

Quadratic equations: factorising

**DEFINITIONS****Quadratic equation**

An equation that can be written $ax^2 + bx + c = 0$ with $a \neq 0$.

Zero-product principle

If two things multiplied together give zero, AT LEAST ONE of them must be zero: if $PQ = 0$ then $P = 0$ or $Q = 0$.

Roots

The x -values that satisfy the equation. A quadratic has up to TWO real roots.

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KEY FORMULAS

- **Standard form:** $ax^2 + bx + c = 0$
- **Step 1:** Get one side = 0
- **Step 2:** Factorise the quadratic
- **Step 3:** Apply the zero-product principle to each factor

WORKED EXAMPLES

Example 1: Solve $x^2 - 7x + 10 = 0$.

- Find two numbers $\times 10$, $+(-7)$: -5 and -2 .

$$-5 \times -2 = 10, -5 + -2 = -7.$$

► $(x - 5)(x - 2) = 0$

Factorise.

► $x - 5 = 0 \rightarrow x = 5$; $x - 2 = 0 \rightarrow x = 2$

Zero-product.

► **$x = 5$ or $x = 2$**

Example 2: Solve $x^2 + x - 12 = 0$.

- $\times(-12)$, $+(1)$: 4 and -3 .

$$4 \times -3 = -12, 4 + (-3) = 1.$$

► $(x + 4)(x - 3) = 0$

► **$x = -4$ or $x = 3$**

Example 3: Solve $x^2 - 4x = 0$.

► $x(x - 4) = 0$

Common factor x .

► **$x = 0$ or $x = 4$**

Zero-product.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. Solve $x^2 - 5x + 6 = 0$.
2. Solve $x^2 + 3x - 10 = 0$.
3. Solve $x^2 - 7x + 12 = 0$.
4. Solve $x^2 + x - 6 = 0$.
5. Solve $x^2 - x - 12 = 0$.
6. Solve $x^2 + 2x = 0$.
7. Solve $x^2 - 4x = 0$.
8. The product of two consecutive integers is 30. Find them.
9. Solve $x^2 + 6x + 9 = 0$.
10. A rectangle of area 20 has length 1 more than width. Find the width.

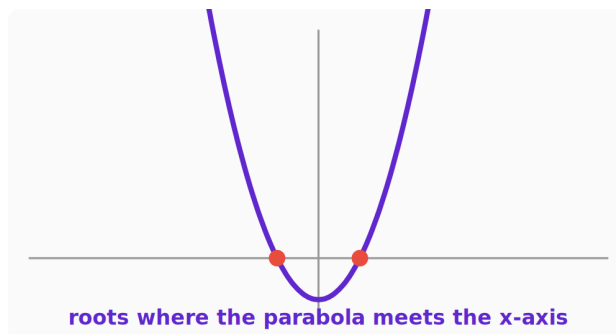


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4.3 Quadratic with difference of squares

Quadratic: difference of squares



DEFINITIONS

Difference of squares

$a^2 - b^2 = (a + b)(a - b)$. When set to zero, gives roots $a = b$ and $a = -b$.

KEY FORMULAS

- **Pattern:** $x^2 - k^2 = 0 \implies (x - k)(x + k) = 0$
- **Roots:** $x = k$ or $x = -k$

WORKED EXAMPLES

Example 1: Solve $x^2 - 49 = 0$.

► $(x - 7)(x + 7) = 0$

$49 = 7^2$, so use difference of squares.

► **$x = 7$ or $x = -7$**

Example 2: Solve $x^2 = 81$.

► $x^2 - 81 = 0$

Move 81 across so RHS = 0.

► $(x - 9)(x + 9) = 0$

$81 = 9^2$.

► **$x = \pm 9$**

EXERCISES (EASY → DIFFICULT)

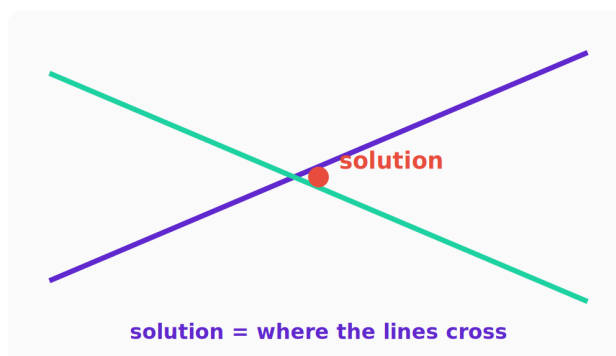
1. Solve $x^2 - 9 = 0$.
2. Solve $x^2 - 25 = 0$.
3. Solve $4x^2 - 1 = 0$.
4. Solve $x^2 - 16 = 0$.
5. Solve $9x^2 - 4 = 0$.
6. Solve $x^2 = 49$.
7. Solve $2x^2 - 8 = 0$.
8. Solve $x^2 - 100 = 0$.
9. A square has area 36 cm^2 . Find the side (positive).
10. Solve $16x^2 - 9 = 0$.



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4.4 Solving simultaneous equations



DEFINITIONS

Simultaneous equations	Two or more equations that must hold AT THE SAME TIME for the same set of unknowns.
Substitution	Make one variable the subject in one equation, then substitute into the other.
Elimination	Add or subtract the equations to remove one variable.

KEY FORMULAS

- **Substitution:** from one equation, write $y = \dots$ and substitute into the other
- **Elimination:** if coefficients of one variable match, add/subtract the equations to cancel it
- **Result:** a one-variable equation — solve, then back-substitute

WORKED EXAMPLES

Example 1: Solve $y = 2x - 1$ and $3x + y = 14$.

► Sub: $3x + (2x - 1) = 14$

Replace y.

► $5x - 1 = 14$, so $x = 3$

► $y = 2(3) - 1 = 5$

Back-substitute.

► **$x = 3, y = 5$**

Check both equations

Example 2: Solve $x + y = 10$ and $x - y = 4$ (elimination).

► Add the equations: $2x = 14$

y-terms cancel.

► $x = 7$

► From $x + y = 10$: $y = 3$

► **$x = 7, y = 3$**

EXERCISES (EASY → DIFFICULT)

1. Solve $y = x + 1$ and $y = 2x - 1$.
2. Solve $x + y = 7$ and $x - y = 1$.
3. Solve $2x + y = 8$ and $y = x - 1$.
4. Solve $x + y = 10$ and $2x + y = 16$.
5. Solve $3x + 2y = 12$ and $x = 2$.



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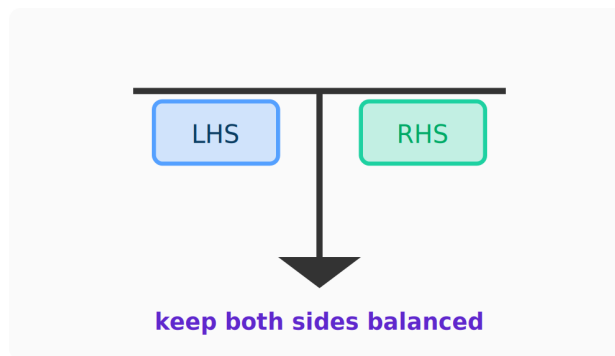
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6. Two numbers sum to 12 and differ by 4. Find them.
7. Solve $x + 2y = 11$ and $x = y + 2$.
8. Tickets: 2 adult + 1 child = R80; adult = R30. Find child price.
9. Solve $4x - y = 5$ and $x + y = 5$.
10. A shop sells pens (x) and books (y): $x + y = 9$ and $2x + 5y = 30$. Find x, y .

4.5 Word problems

Word problems with linear equations



DEFINITIONS

Translation	Converting English sentences into algebra: "is" $\rightarrow$ $=$, "more than" $\rightarrow$ $+$, "twice" $\rightarrow$ $2\times$, etc.
Let statement	A clear sentence saying what each variable stands for. Always start with this.

KEY FORMULAS

- **Step 1:** Read carefully — what is unknown?
- **Step 2:** Let x = the unknown (define units!)
- **Step 3:** Translate into an equation
- **Step 4:** Solve and check the answer in the original wording

WORKED EXAMPLES

Example 1: The sum of three consecutive integers is 72. Find them.

- Let x = the smallest. Then $x, x+1, x+2$.

Consecutive means each is 1 more.

► $x + (x+1) + (x+2) = 72$

Build the equation.

► $3x + 3 = 72$, so $x = 23$

► **23, 24, 25**

Check: $23+24+25 = 72$



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Example 2: A father is 4 times as old as his son. In 5 years he will be only 3 times as old. Find their current ages.

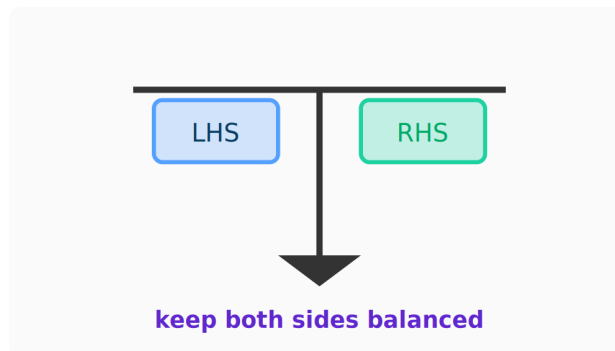
- ▶ Let son = x . Then father = $4x$.
- ▶ In 5 yrs: $4x + 5 = 3(x + 5)$
- ▶ $4x + 5 = 3x + 15 \rightarrow x = 10$
- ▶ **Son 10, father 40**

EXERCISES (EASY → DIFFICULT)

1. A number plus 7 is 19. Find it.
2. Twice a number minus 3 is 11. Find it.
3. Three consecutive integers sum to 36. Find them.
4. A father is $3 \times$ his son's age; together 48. Find the son's age.
5. A rectangle's length is 5 more than width; perimeter 38. Find width.
6. Naledi buys 3 pens at R x each and a R12 book for R27. Find x .
7. The sum of two numbers is 40; one is $4 \times$ the other. Find them.
8. A taxi charges R6 + R4/km; a trip cost R34. Find the distance.
9. A class has b boys and $2b$ girls, totalling 30. Find b .
10. Thabo saves R50 then R20/week; after how many weeks has he R210?

4.6 Literal equations

Literal equations (changing the subject)



DEFINITIONS

Literal equation

An equation involving several letters where you must isolate ONE specific letter.

Subject of the formula

The variable on its own on one side of the formula, e.g. A in $A = \pi r^2$.

KEY FORMULAS

- **Goal:** isolate the named variable on one side
- **Method:** use the same inverse-operation rules as for numerical equations
- **Treat all OTHER letters as constants**



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WORKED EXAMPLES

Example 1: Make r the subject of $C = 2\pi r$.

$$\triangleright C \div (2\pi) = r$$

Divide both sides by 2π .

$$\triangleright r = C / (2\pi)$$

Example 2: Make x the subject of $y = mx + c$.

$$\triangleright y - c = mx$$

Subtract c .

$$\triangleright x = (y - c) / m$$

Divide by m .

EXERCISES (EASY → DIFFICULT)

1. Make x the subject: $y = x + a$.
2. Make x the subject: $y = 2x$.
3. Make x the subject: $y = mx + c$.
4. Make r the subject: $C = 2\pi r$.
5. Make h the subject: $A = \frac{1}{2}bh$.
6. Make x the subject: $ax + b = c$.
7. Make t the subject: $v = u + at$.
8. Make b the subject: $P = 2a + 2b$.
9. Make r the subject: $A = \pi r^2$.
10. Make x the subject: $y = (x - 3)/4$.

4.7 Solve linear inequality

Linear inequalities



open circle: 3 is NOT included
 $x > 3$

DEFINITIONS

Inequality	A statement using $<$, $>$, $\leq$ or $\geq$ instead of $=$. The solution is usually a RANGE of values.
Solution set	All values of x that make the inequality true. Often shown on a number line.

KEY FORMULAS

- Same rules as equations for $+$ and $-$
- Same rules for $\times$ or $\div$ by a POSITIVE number
- FLIP the inequality when $\times$ or $\div$ by a NEGATIVE number



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WORKED EXAMPLES

Example 1: Solve $2x + 3 < 11$.

$$\triangleright 2x < 8$$

Subtract 3.

$$\triangleright x < 4$$

Divide by +2 — sign unchanged.

Example 2: Solve $3x - 5 \geq 7$.

$$\triangleright 3x \geq 12$$

Add 5.

$$\triangleright x \geq 4$$

Divide by +3 — sign unchanged.

EXERCISES (EASY → DIFFICULT)

1. Solve $x + 3 > 7$.
2. Solve $2x < 10$.
3. Solve $x - 4 \geq 1$.
4. Solve $3x + 1 \leq 10$.
5. Solve $2x + 5 > 11$.
6. Solve $(x/2) \geq 4$.
7. Represent $x > 2$ on a number line (describe).
8. Solve $4x - 3 < 9$.
9. A learner needs at least 50% of 80 marks. How many marks?
10. Solve $5x + 2 \leq 17$.

4.7 Inequality with negative coefficient



$$\div \text{ by a negative} \Rightarrow \text{flip } < \text{ to } > \\ -2x < 6 \Rightarrow x > -3$$

DEFINITIONS

Sign-flip rule

When you multiply or divide BOTH sides of an inequality by a negative number, you MUST reverse the inequality sign.

KEY FORMULAS

- If $a < b$ and $k < 0$ then $ka > kb$ (sign flipped!)
- **Example:** from $-2x < 6$, divide by -2 to get $x > -3$



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WORKED EXAMPLES

Example 1: Solve $-2x < 8$.

- Divide both sides by -2 — FLIP the sign.

Negative divisor!

► $x > -4$

Check: try $x = 0$: $-2(0) = 0 < 8$

Example 2: Solve $5 - 3x \geq 14$.

► $-3x \geq 9$

Subtract 5.

► Divide by -3 , flip sign $\rightarrow x \leq -3$

Negative divisor flips $\geq$ to $\leq$.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. Solve $-x > 3$.
2. Solve $-2x < 8$.
3. Solve $-3x \geq 9$.
4. Solve $5 - x > 2$.
5. Solve $-x/2 \leq 4$.
6. Solve $1 - 2x \geq 7$.
7. Why does the inequality sign flip here?
8. Solve $3 - 4x < 11$.
9. Solve $-5x + 2 \leq -13$.
10. Solve $10 - 3x > 4$.

End of Chapter 4 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Solve $x + 5 = 12$.
2. Solve $3x - 5 = 16$.
3. Solve $4(2x - 1) = 20$.
4. Solve $7x - 2 = 4x + 10$.
5. Solve $x^2 - x - 12 = 0$.
6. Solve $x^2 = 49$.
7. Solve $x + 2y = 11$ and $x = y + 2$.
8. A taxi charges R6 + R4/km; a trip cost R34. Find the distance.
9. Make r the subject: $A = \pi r^2$.
10. Solve $5x + 2 \leq 17$.



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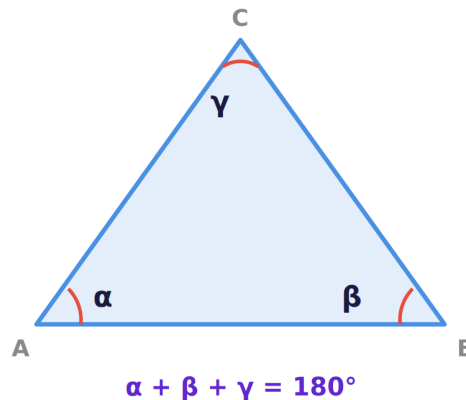
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CHAPTER 7

Euclidean geometry

- 7.2 Triangle angle sum
- 7.2 Exterior angle of a triangle
- 7.2 Congruency tests (SSS, SAS, AAS, RHS)
- 7.2 Similarity tests (AA, SAS, SSS-proportional)
- 7.3 Quadrilaterals
- 7.4 Mid-point theorem

7.2 Triangle angle sum



DEFINITIONS

- Interior angles** The three angles inside a triangle, where the sides meet.
- Angle sum** For ANY triangle, the three interior angles add up to 180° .

KEY FORMULAS

- **Sum of angles in a triangle:** $\angle A + \angle B + \angle C = 180^\circ$
- **Find a missing angle:** $180^\circ - (\text{sum of the other two})$



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WORKED EXAMPLES

Example 1: Two angles of a triangle are 50° and 70° . Find the third.

► $180^\circ - 50^\circ - 70^\circ = 60^\circ$

Subtract from 180° .

► **Third angle = 60°**

Example 2: A right-angled triangle has a 35° angle. Find the third.

► Right angle = 90°

► $180^\circ - 90^\circ - 35^\circ = 55^\circ$

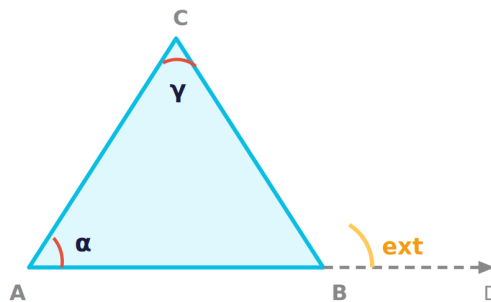
► **Third = 55°**

EXERCISES (EASY → DIFFICULT)

- Two angles of a triangle are 50° and 60° . Find the third.
- A right triangle has one angle 35° . Find the third.
- Each angle of an equilateral triangle is...
- Angles are x , x , and 80° (isosceles). Find x .
- Angles are $2x$, $3x$, $4x$. Find x .
- An isosceles triangle has apex 40° . Find each base angle.
- Angles $(x + 10)$, $(x + 20)$, $(x + 30)$. Find x .
- A roof truss triangle has angles in ratio 1:2:3. Find the largest.
- In $\triangle ABC$, $\angle A = 90^\circ$, $\angle B = 2\angle C$. Find $\angle C$.
- A triangular sail has two equal angles and a 100° angle. Find each equal angle.

7.2 Exterior angle of triangle

Exterior angle of a triangle



$$\text{ext} = \alpha + \gamma \text{ (sum of two opposite interior)}$$

DEFINITIONS

Exterior angle The angle formed when ONE side of the triangle is extended outwards.

Remote interior angles The two interior angles NOT next to the extended side.



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KEY FORMULAS

- **Exterior angle theorem:** exterior $\angle$ = sum of the two REMOTE interior angles
- **Adjacent angles:** interior + exterior = 180° (straight line)

WORKED EXAMPLES

Example 1: Two interiors of a triangle are 40° and 65° . Find the exterior at the third vertex.

- ▶ Exterior = $40^\circ + 65^\circ$
Sum of remote interiors.
- ▶ = 105°

Example 2: A triangle has a 30° angle. The exterior at one of the other vertices is 100° . Find both remaining interiors.

- ▶ Exterior $100^\circ = 30^\circ +$ (other interior)
- ▶ Other interior at remote vertex = 70°
- ▶ Interior at the vertex with the 100° exterior = 80°
 $180^\circ - 100^\circ$.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

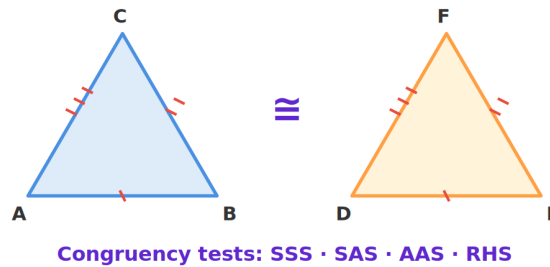
1. An exterior angle of a triangle is 120° ; one remote interior angle is 50° . Find the other.
2. Exterior angle = sum of the two...
3. Remote interior angles are 45° and 55° . Find the exterior angle.
4. Exterior angle 110° , one remote interior 60° . Find the other.
5. An equilateral triangle's exterior angle is...
6. Exterior angle $(2x)$, remote interior 40° and 50° . Find x .
7. A triangle's exterior angle is 130° . Find the adjacent interior angle.
8. Remote interior angles 70° and (x) . Exterior angle 115° . Find x .
9. Why does the exterior-angle rule hold?
10. Exterior angle 95° , remote interiors equal. Find each.



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7.2 Congruency tests (SSS, SAS, AAS, RHS)



DEFINITIONS

Congruent	Two figures are congruent if they have EXACTLY the same shape AND size — one can be mapped onto the other by translation, rotation or reflection.
Test	A minimum set of equal parts proving two triangles are congruent.

KEY FORMULAS

- **SSS:** all three pairs of SIDES equal
- **SAS:** two sides AND the included angle equal
- **AAS** (or ASA): two angles AND a corresponding side equal
- **RHS:** Right angle, Hypotenuse and one Side equal

WORKED EXAMPLES

Example 1: $\triangle ABC$ has $AB=5$, $BC=7$, $AC=8$. $\triangle PQR$ has $PQ=5$, $QR=7$, $PR=8$. Are they congruent?

- All 3 pairs of sides equal.
- Yes — by SSS

Example 2: $\triangle ABC$: $\angle A=90^\circ$, $AB=6$, $BC=10$ (hypotenuse). $\triangle XYZ$: $\angle X=90^\circ$, $XY=6$, $YZ=10$. Congruent?

- Right angle, hypotenuse 10, one side 6 — match.
- Yes — by RHS

EXERCISES (EASY → DIFFICULT)

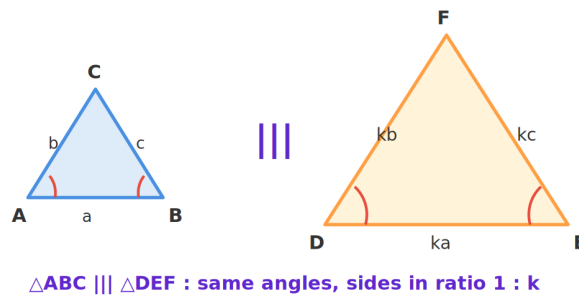
- Which test: three pairs of equal sides?
- Which test: two sides and the included angle equal?
- Which test: two angles and a corresponding side?
- Which test: right angle, hypotenuse, and a side?
- Are two triangles with all sides 3, 4, 5 congruent?
- $\triangle ABC$ and $\triangle DEF$: $AB = DE$, $\angle A = \angle D$, $AC = DF$. Which test?
- Right triangles with equal hypotenuse and one equal leg — congruent?
- If two angles and the included side are equal, which test applies?
- Do equal angles alone (AAA) prove congruency?
- State why SAS works but SSA generally does not.



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7.2 Similarity tests (AA, SAS, SSS-proportional)



DEFINITIONS

Similar

Two figures are similar if they have the SAME SHAPE but possibly different sizes — corresponding angles are equal AND corresponding sides are in proportion.

Scale factor (k)

The ratio of corresponding sides between two similar figures. If $k = 2$, the second figure is twice as big.



The symbol meaning "is similar to". $\triangle ABC \parallel \triangle DEF$ reads "triangle ABC is similar to triangle DEF".

KEY FORMULAS

- **AA (or AAA):** two pairs of corresponding angles equal
- **SAS (proportional):** two pairs of corresponding sides in proportion AND the included angles equal
- **SSS (proportional):** three pairs of corresponding sides in proportion
- **Side ratio:** $AB/DE = BC/EF = CA/FD = k$ (the scale factor)

WORKED EXAMPLES

Example 1: $\triangle ABC$ has angles 50° , 60° , 70° . $\triangle PQR$ has angles 50° , 60° , 70° . Are they similar?

- Two pairs of angles match (in fact all three).

AA test only needs two pairs equal; the third matches automatically because angles sum to 180° .

- **Yes — $\triangle ABC \parallel \triangle PQR$ by AA**

Same shape; sizes may differ.

Example 2: In $\triangle ABC$ and $\triangle DEF$, $AB = 4$, $BC = 6$, $AC = 8$. $DE = 6$, $EF = 9$, $DF = 12$. Are they similar?

- Check ratios: $4/6 = 2/3$, $6/9 = 2/3$, $8/12 = 2/3$.

Form the ratios of corresponding sides.

- All three ratios equal $2/3$.

Sides are in proportion with $k = 3/2$ (DEF is $1.5 \times ABC$).

- **Yes — $\triangle ABC \parallel \triangle DEF$ by SSS**

The SSS-proportional similarity test.



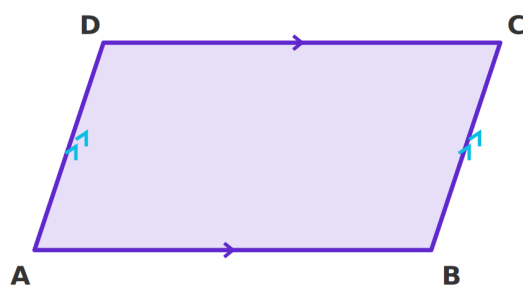
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EXERCISES (EASY → DIFFICULT)

1. Which similarity test: two pairs of equal angles?
2. Which test: three pairs of sides in proportion?
3. Which test: two sides in proportion and included angle equal?
4. Similar triangles have equal... and sides in...
5. Triangle sides 3,4,5 and 6,8,10 — similar?
6. A 2 m pole casts 3 m shadow; a tree casts 18 m. Find the tree height.
7. Two triangles share two equal angles — similar?
8. Similar triangles, ratio 1:3; small area 5. Find the large area.
9. A photo 4×6 is enlarged to width 12. Find the new length.
10. Map scale 1:50 000; 4 cm on map = ? km real.

7.3 Quadrilaterals



Parallelogram: $AB \parallel DC$, $AD \parallel BC$

DEFINITIONS

Quadrilateral	A four-sided polygon. The interior angles add to 360° .
Parallelogram	Both pairs of opposite sides parallel; opposite sides and angles equal; diagonals bisect each other.
Rectangle	A parallelogram with all angles 90° . Diagonals are equal.
Rhombus	A parallelogram with all sides equal. Diagonals bisect at 90° and bisect the angles.
Square	A rectangle AND a rhombus — all sides equal, all angles 90° .
Trapezium	A quadrilateral with at LEAST one pair of parallel sides.

KEY FORMULAS

- **Sum of interior angles:** 360°
- **Parallelogram:** opposite sides equal & parallel; opposite angles equal
- **Rectangle:** all the above + diagonals equal
- **Rhombus:** all sides equal; diagonals perpendicular & bisect each other



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WORKED EXAMPLES

Example 1: ABCD is a parallelogram. $\angle A = 70^\circ$. Find $\angle B$, $\angle C$, $\angle D$.

► Opposite angles equal: $\angle C = 70^\circ$

Property of parallelogram.

► Co-interior: $\angle A + \angle B = 180^\circ$

Adjacent angles.

► $\angle B = 110^\circ$, $\angle C = 70^\circ$, $\angle D = 110^\circ$

Example 2: A quadrilateral has angles 80° , 95° , 110° . Find the fourth.

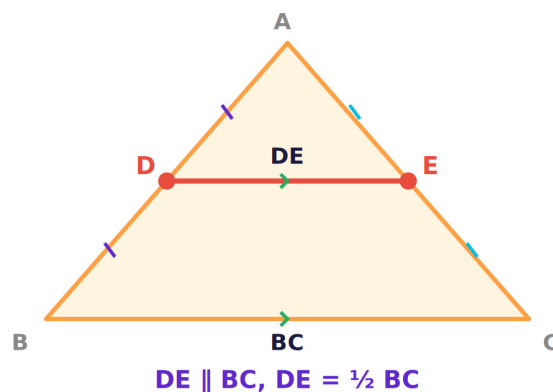
► $360^\circ - 80^\circ - 95^\circ - 110^\circ = 75^\circ$

► **Fourth angle = 75°**

EXERCISES (EASY → DIFFICULT)

1. The angles of a quadrilateral sum to...
2. Three angles are 90° , 90° , 100° . Find the fourth.
3. Opposite angles of a parallelogram are...
4. A rectangle's diagonals are...
5. A rhombus has all sides...
6. Angles of a quad: x , $2x$, $3x$, $4x$. Find x .
7. Co-interior angles of a parallelogram sum to...
8. A square is a rhombus with...
9. A parallelogram has one angle 70° . Find the others.
10. Diagonals of a rhombus meet at...

7.4 Mid-point theorem



DEFINITIONS

Mid-point

The point that divides a line segment into two EQUAL halves.

Mid-point theorem

In any triangle, the line joining the mid-points of two sides is PARALLEL to the third side and HALF its length.



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KEY FORMULAS

- **Mid-point theorem:** if M, N are mid-points of AB, AC then $MN \parallel BC$ and $MN = \frac{1}{2} BC$
- **Converse:** a line through one mid-point parallel to a side bisects the third side

WORKED EXAMPLES

Example 1: In $\triangle ABC$, M is the mid-point of AB, N is the mid-point of AC, $BC = 12$ cm. Find MN.

► $MN = \frac{1}{2} \times BC$

Mid-point theorem.

► **$MN = 6$ cm**

Example 2: M is the mid-point of AB. A line through M parallel to BC meets AC at N. If $AC = 14$ cm, find AN.

► By converse: N is the mid-point of AC.

► **$AN = 7$ cm**

EXERCISES (EASY → DIFFICULT)

1. The mid-point theorem: the segment joining mid-points of two sides is... the third side.
2. A triangle's third side is 12. Find the mid-segment length.
3. Mid-segment = 7. Find the third side.
4. Mid-points of a side: if the side is 10, half is...
5. The mid-segment is parallel to which side?
6. In $\triangle ABC$, D and E are mid-points of AB and AC; $BC = 16$. Find DE.
7. DE (mid-segment) = 9; $BC = ?$
8. A triangular frame's base is 20 cm; a brace joins the two mid-points. Find its length.
9. Why is the mid-segment half the base?
10. Mid-segment $DE \parallel BC$ and $DE = 6.5$. Find BC.

End of Chapter 7 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Two angles of a triangle are 50° and 60° . Find the third.
2. Exterior angle = sum of the two...
3. Which test: two angles and a corresponding side?
4. Similar triangles have equal... and sides in...
5. A rhombus has all sides...
6. In $\triangle ABC$, D and E are mid-points of AB and AC; $BC = 16$. Find DE.
7. Angles $(x + 10)$, $(x + 20)$, $(x + 30)$. Find x.
8. Remote interior angles 70° and (x) . Exterior angle 115° . Find x.
9. Do equal angles alone (AAA) prove congruency?
10. Map scale 1:50 000; 4 cm on map = ? km real.





CHAPTER 8

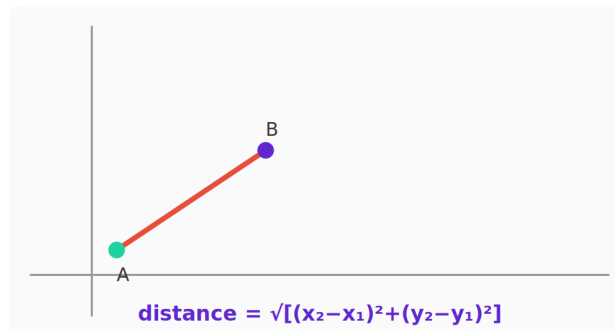
Analytical geometry

8.2 Distance between two points

8.3 Gradient of a line

8.4 Mid-point of a line segment

8.2 Distance between two points



DEFINITIONS

Coordinate	A pair (x, y) describing a point's position on the Cartesian plane.
Distance formula	Pythagoras applied on the Cartesian plane to find the length of the line segment between two points.

KEY FORMULAS

- **Distance:** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- **Comes from Pythagoras:** horizontal Δx and vertical Δy form a right triangle whose hypotenuse is d

WORKED EXAMPLES

Example 1: Find the distance between A(1, 2) and B(4, 6).

- ▶ $\Delta x = 4 - 1 = 3$, $\Delta y = 6 - 2 = 4$
- ▶ $d = \sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = \sqrt{25}$
Pythagorean triple!
- ▶ **$d = 5$ units**



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Example 2: Distance between P(-2, 3) and Q(4, -5).

$$\triangleright \Delta x = 4 - (-2) = 6, \Delta y = -5 - 3 = -8$$

Mind the signs.

$$\triangleright d = \sqrt{(36 + 64)} = \sqrt{100}$$

$$\triangleright d = 10 \text{ units}$$

Example 3: Show that A(0, 0), B(3, 4), C(0, 5) form a right triangle.

$$\triangleright AB = \sqrt{(9+16)} = 5$$

$$\triangleright BC = \sqrt{(9+1)} = \sqrt{10}, AC = 5$$

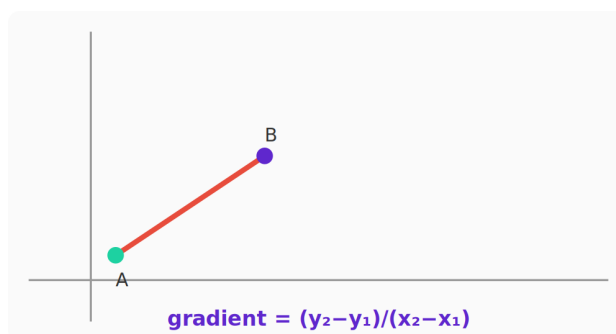
$$\triangleright AB^2 + (\text{something}) \text{ test... here } AC^2 + AB^2 = 25 + 25 \neq BC^2. \text{ Re-check.}$$

Use the longest side as hypotenuse and test.

EXERCISES (EASY → DIFFICULT)

- Find the distance between (0, 0) and (3, 4).
- Find the distance between (1, 2) and (4, 6).
- Find the distance between (0, 0) and (5, 12).
- Find the distance between (-1, 0) and (2, 4).
- Find the distance between (2, 3) and (2, 9).
- On a town grid (km), a school is at (1, 1) and a clinic at (4, 5). How far apart?
- A drone flies from (0, 0) to (6, 8) km. How far?
- Find the distance between (-2, -1) and (1, 3).
- Two towers at (0, 0) and (8, 6) (km). Find the distance.
- Show (0,0), (3,4), (6,8) are collinear using distances or gradient.

8.3 Gradient of a line


DEFINITIONS

Gradient (slope)	A measure of how steep a line is — the rise (change in y) divided by the run (change in x).
Positive gradient	Line rises from left to right.
Negative gradient	Line falls from left to right.
Zero gradient	Horizontal line (no rise).
Undefined gradient	Vertical line (run = 0; division by zero).


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KEY FORMULAS

- **Gradient:** $m = (y_2 - y_1) / (x_2 - x_1)$
- **Memory:** rise over run
- **Parallel lines** have EQUAL gradients
- **Perpendicular lines:** $m_1 \cdot m_2 = -1$

WORKED EXAMPLES

Example 1: Find the gradient of the line through (2, 5) and (6, 13).

► $m = (13 - 5) / (6 - 2)$

► $m = 8 / 4$

► **$m = 2$**

Line rises 2 units for every 1 unit right.

Example 2: Gradient of line through (-1, 4) and (3, -4).

► $m = (-4 - 4) / (3 - (-1)) = -8/4$

► **$m = -2$**

Negative — line falls.

Example 3: Gradient of horizontal line through (1, 5) and (7, 5).

► $m = (5 - 5) / (7 - 1) = 0/6$

► **$m = 0$**

Horizontal line has zero gradient.

EXERCISES (EASY → DIFFICULT)

1. Find the gradient through (0, 0) and (2, 6).
2. Find the gradient through (1, 2) and (3, 8).
3. Find the gradient through (0, 4) and (2, 0).
4. Find the gradient through (1, 1) and (5, 9).
5. A horizontal line has gradient...
6. A straight road rises from (0, 0) to (4, 2). Find its gradient.
7. Find the gradient through (-2, 3) and (2, 11).
8. Parallel lines have gradients that are...
9. A ramp goes from (0, 0) to (5, 1). Find its gradient.
10. Perpendicular lines have gradients whose product is...

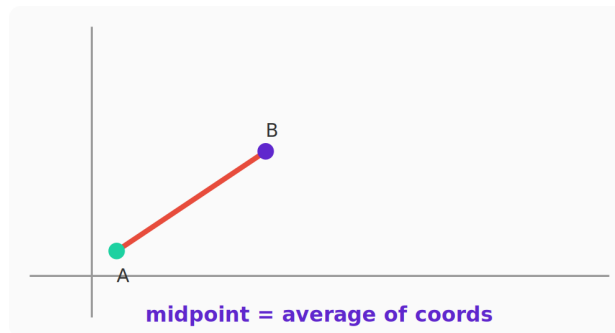


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8.4 Mid-point of a line

Mid-point of a line segment



DEFINITIONS

Mid-point

The point exactly halfway along a line segment — the AVERAGE of the two endpoints.

KEY FORMULAS

- **Mid-point:** $M = ((x_1 + x_2) / 2 , (y_1 + y_2) / 2)$
- **Average the x's, average the y's**

WORKED EXAMPLES

Example 1: Mid-point of A(2, 4) and B(8, 10).

- ▶ $x_M = (2 + 8) / 2 = 5$
- ▶ $y_M = (4 + 10) / 2 = 7$
- ▶ **M = (5, 7)**

Example 2: Mid-point of A(-3, 1) and B(5, -7).

- ▶ $x_M = (-3 + 5) / 2 = 1$
- ▶ $y_M = (1 + (-7)) / 2 = -3$
- ▶ **M = (1, -3)**

EXERCISES (EASY → DIFFICULT)

- Find the mid-point of (2, 4) and (6, 8).
- Find the mid-point of (0, 0) and (4, 10).
- Find the mid-point of (-2, 3) and (4, -1).
- Find the mid-point of (1, 1) and (7, 5).
- Find the mid-point of (-3, -2) and (3, 2).
- A fence runs from (1, 2) to (5, 8); where is its midpoint gate?
- Find the mid-point of (0, 5) and (0, -5).
- The midpoint of A(2, y) and B(8, 10) is (5, 7). Find y.
- A road from (-4, 0) to (4, 6) has a rest stop at the midpoint. Where?
- M(3, 4) is the midpoint of P(1, 2) and Q. Find Q.



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End of Chapter 8 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Find the distance between $(0, 0)$ and $(3, 4)$.
2. Find the gradient through $(1, 2)$ and $(3, 8)$.
3. Find the mid-point of $(-2, 3)$ and $(4, -1)$.
4. Find the distance between $(-1, 0)$ and $(2, 4)$.
5. A horizontal line has gradient...
6. A fence runs from $(1, 2)$ to $(5, 8)$; where is its midpoint gate?
7. A drone flies from $(0, 0)$ to $(6, 8)$ km. How far?
8. Parallel lines have gradients that are...
9. A road from $(-4, 0)$ to $(4, 6)$ has a rest stop at the midpoint. Where?
10. Show $(0,0)$, $(3,4)$, $(6,8)$ are collinear using distances or gradient.

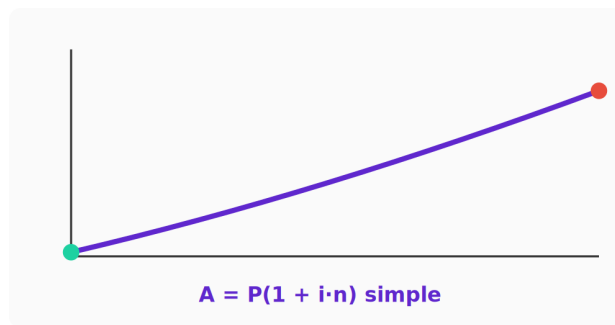


CHAPTER 9

Finance and growth

- 9.2 Simple interest
- 9.3 Compound interest
- 9.4 Choosing simple vs compound
- 9.5 Foreign exchange rates

9.2 Simple interest



DEFINITIONS

Principal (P)	The original amount of money invested or borrowed.
Interest rate (i)	The percentage charged or earned, usually per YEAR.
Time (n)	How long the money is invested or borrowed, usually in YEARS.
Simple interest	Interest calculated only on the ORIGINAL principal — does not earn interest on previous interest.

KEY FORMULAS

- **Simple interest formula:** $A = P(1 + i \cdot n)$
- **Interest only:** $I = P \cdot i \cdot n$
- **Convert percentage:** $i = \text{rate}\% \div 100$ (e.g. 8% → 0.08)

WORKED EXAMPLES



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**Example 1: R5 000 invested at 7% simple interest p.a. for 4 years. Find A.**

$$\triangleright P = 5000, i = 0.07, n = 4$$

Convert 7% $\rightarrow 0.07$.

$$\triangleright A = 5000(1 + 0.07 \times 4) = 5000(1.28)$$

$$\triangleright A = \text{R6 400}$$

Example 2: How much interest does R8 000 earn at 6% simple interest over 5 years?

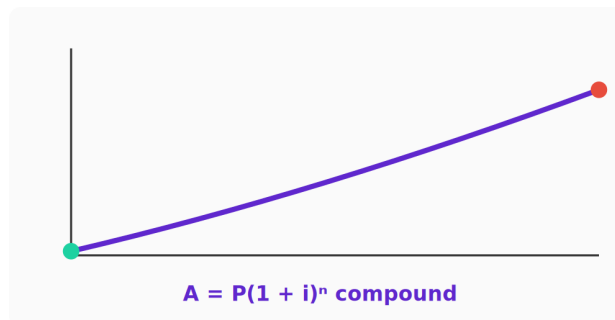
$$\triangleright I = P \times i \times n = 8000 \times 0.06 \times 5$$

$$\triangleright I = \text{R2 400}$$

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. Find the simple interest on R1 000 at 8% p.a. for 1 year.
2. Find the simple interest on R2 000 at 10% p.a. for 3 years.
3. Find the value of R5 000 at 6% p.a. simple interest after 2 years.
4. Find the simple interest on R800 at 5% p.a. for 4 years.
5. Simple interest formula: $A = ?$
6. Thabo invests R12 000 at 7,5% p.a. simple interest for 5 years. Find the value.
7. How long for R1 000 to earn R240 at 8% p.a. simple interest?
8. Lerato borrows R4 000 at 12% p.a. simple interest. Find the interest after 2 years.
9. At what rate does R2 000 grow to R2 300 in 2 years (simple interest)?
10. A R6 000 loan at 9% p.a. simple interest is repaid after 3 years. Find the total repaid.

9.3 Compound interest

**DEFINITIONS****Compound interest**

Interest is added to the principal at the end of each period, then **EARNs** INTEREST itself in the next period — interest on interest.

Compounding period

How often interest is added: yearly, monthly, etc. More frequent = faster growth.

KEY FORMULAS

- **Compound interest formula:** $A = P(1 + i)^n$
- **Where:** P = principal, i = rate per period (decimal), n = number of periods
- **Growth:** EXPONENTIAL — much faster than simple interest over time

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WORKED EXAMPLES

Example 1: R5 000 at 7% compound interest p.a. for 4 years.

$$\triangleright A = 5000(1.07)^4$$

$$\triangleright 1.07^4 \approx 1.3108$$

Use calculator.

$$\triangleright A \approx \mathbf{R6\ 553.98}$$

Compare to R6 400 simple — already R154 more.

Example 2: R10 000 at 5% compound interest for 10 years.

$$\triangleright A = 10000(1.05)^{10}$$

$$\triangleright (1.05)^{10} \approx 1.6289$$

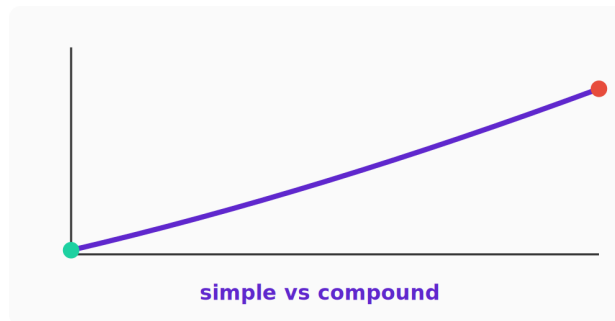
$$\triangleright A \approx \mathbf{R16\ 288.95}$$

EXERCISES (EASY → DIFFICULT)

1. Find the value of R1 000 at 8% p.a. compound interest after 2 years.
2. Find the value of R2 000 at 10% p.a. compounded annually after 3 years.
3. Compound interest formula: $A = ?$
4. Find the value of R5 000 at 6% p.a. compounded annually after 4 years.
5. Find the interest earned on R3 000 at 7% p.a. compounded for 2 years.
6. Naledi invests R8 000 at 9% p.a. compounded annually for 5 years. Find the value.
7. R10 000 grows at 8% p.a. compounded annually. Find its value after 3 years.
8. Compare R1 000 at 10% simple vs compound after 2 years.
9. R4 000 at 12% p.a. compounded annually — value after 2 years?
10. How much must Sipho invest now at 8% p.a. compound to have $\approx$ R5 832 in 2 years?

9.4 Calculations with simple/compound interest

Choosing simple vs compound



DEFINITIONS

Linear growth	Equal absolute increase per period — gives a STRAIGHT-LINE graph (simple interest).
Exponential growth	Equal PERCENTAGE increase per period — gives a CURVED graph getting steeper (compound interest).



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KEY FORMULAS

- **Simple:** $A = P(1 + i \cdot n)$ — used when interest is paid out each period
- **Compound:** $A = P(1 + i)^n$ — used when interest is REINVESTED
- **Compound > Simple** for $n > 1$ year (and any $i > 0$)

WORKED EXAMPLES

Example 1: R10 000 at 8% for 10 years — find the difference between simple and compound.

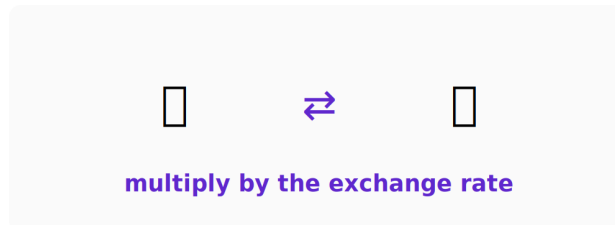
- ▶ Simple: $A = 10000(1 + 0.08 \times 10) = 10000(1.8) = \text{R}18\,000$
- ▶ Compound: $A = 10000(1.08)^{10} \approx 10000(2.1589) \approx \text{R}21\,589$
- ▶ **Difference $\approx \text{R}3\,589$**

Compound wins by a lot.

EXERCISES (EASY → DIFFICULT)

1. R2 000 at 9% p.a. simple interest for 4 years — final value?
2. R2 000 at 9% p.a. compound for 4 years — final value?
3. Which earns more over time: simple or compound interest?
4. R15 000 at 7% p.a. compounded annually for 6 years.
5. Find the interest only on R10 000 at 6% p.a. compounded for 3 years.
6. Thabo saves R20 000 at 8,5% p.a. compounded annually. Value after 10 years?
7. A R50 000 car loan at 11% p.a. simple interest over 5 years — total repaid?
8. R1 000 doubles at 10% p.a. compound after about how many years?
9. R7 500 at 6,4% p.a. compounded annually for 3 years.
10. Lerato compares R5 000 at 7% simple (3 yr) vs 7% compound (3 yr).

9.5 Foreign exchange rates



DEFINITIONS

Exchange rate	How much of one currency you get for one unit of another, e.g. 1 USD = R18.50.
Buying rate	The rate at which a bank BUYS foreign currency from you (always less favourable to you).
Selling rate	The rate at which a bank SELLS foreign currency to you.



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KEY FORMULAS

- **Convert FROM rand:** amount in foreign = $\text{ZAR} \div \text{rate}$
- **Convert TO rand:** amount in ZAR = $\text{foreign} \times \text{rate}$
- **Always check WHICH currency the rate is for**

WORKED EXAMPLES

Example 1: 1 USD = R18.50. Convert R3 700 to USD.

► $\text{USD} = \text{R} \div \text{rate} = 3700 \div 18.50$

► = **200 USD**

Example 2: 1 GBP = R23.00. Convert £150 to ZAR.

► $\text{ZAR} = 150 \times 23.00$

Foreign $\times$ rate.

► = **R3 450**

EXERCISES (EASY → DIFFICULT)

1. R1 = \$0.055. How many dollars is R100?
2. \$1 = R18. How many rand is \$50?
3. £1 = R23. Convert £20 to rand.
4. €1 = R20. Convert R400 to euro.
5. \$1 = R18. Convert R1 800 to dollars.
6. A jacket costs \$40. At \$1 = R19, find the rand price.
7. Naledi exchanges R2 700 at £1 = R27. How many pounds?
8. A tourist changes \$200 at \$1 = R18.50. How many rand?
9. If \$1 = R18 and €1 = R20, how many euro is \$90?
10. A phone costs \$300; shipping adds 10%. At \$1 = R19, find the rand total.

End of Chapter 9 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Find the simple interest on R1 000 at 8% p.a. for 1 year.
2. Find the value of R2 000 at 10% p.a. compounded annually after 3 years.
3. Which earns more over time: simple or compound interest?
4. €1 = R20. Convert R400 to euro.
5. Simple interest formula: $A = ?$
6. Naledi invests R8 000 at 9% p.a. compounded annually for 5 years. Find the value.
7. A R50 000 car loan at 11% p.a. simple interest over 5 years — total repaid?
8. A tourist changes \$200 at \$1 = R18.50. How many rand?
9. At what rate does R2 000 grow to R2 300 in 2 years (simple interest)?
10. How much must Sipho invest now at 8% p.a. compound to have $\approx$ R5 832 in 2 years?



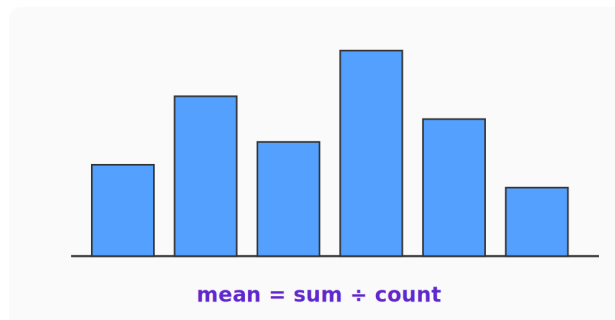
CHAPTER 10

Statistics

- 10.2** Mean (average)
- 10.2** Median (middle value)
- 10.2** Mode (most frequent)
- 10.3** Grouping data (frequency tables)
- 10.4** Range
- 10.4** Inter-quartile range (IQR)
- 10.5** Five-number summary

10.2 Mean

Mean (average)



DEFINITIONS

Mean	The arithmetic average — the sum of all values divided by how many values there are.
When to use	Best for symmetric data with no extreme outliers.

KEY FORMULAS

- **Mean:** $\bar{x} = (\text{sum of all values}) \div (\text{number of values})$
- $\bar{x} = \Sigma x / n$
- **For equal weights:** add them all, divide by how many



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WORKED EXAMPLES

Example 1: Find the mean of 4, 7, 9, 12, 8.

- ▶ Sum = $4 + 7 + 9 + 12 + 8 = 40$
- ▶ $n = 5$
- ▶ Mean = $40 \div 5 = 8$

Example 2: Test marks: 65, 70, 80, 95. Find the mean.

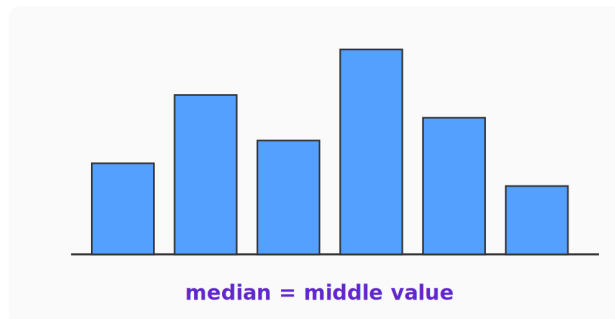
- ▶ Sum = 310
- ▶ Mean = $310 \div 4 = 77.5$

EXERCISES (EASY → DIFFICULT)

1. Find the mean of 4, 6, 8.
2. Find the mean of 2, 4, 6, 8.
3. Find the mean of 10, 20, 30, 40, 50.
4. Find the mean of 3, 7, 5, 9.
5. Find the mean of 12, 15, 18.
6. Test marks: 60, 70, 80, 90. Find the mean.
7. Daily sales (R): 200, 250, 300. Find the mean.
8. Five learners scored 8, 6, 7, 9, 10. Find the mean.
9. The mean of 4 numbers is 12. Find their total.
10. Three numbers have mean 10; two are 8 and 9. Find the third.

10.2 Median

Median (middle value)



DEFINITIONS

Median

The MIDDLE value when the data is arranged in order. For an even count, the average of the two middle values.

Why use it

The median is NOT affected by outliers — better than mean for skewed data.

KEY FORMULAS

- **Step 1:** Sort the data smallest → largest
- **Step 2:** Find the middle position
- **If n is odd:** median is the $(n+1)/2$ th value
- **If n is even:** median = average of the $n/2$ th and $(n/2 + 1)$ th values



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WORKED EXAMPLES

Example 1: Find the median of 3, 8, 5, 12, 7.

- ▶ Sort: 3, 5, 7, 8, 12
- ▶ $n = 5$ (odd), middle position = 3
- ▶ **Median = 7**

Middle of the sorted list.

Example 2: Find the median of 4, 9, 2, 8, 11, 5.

- ▶ Sort: 2, 4, 5, 8, 9, 11
- ▶ $n = 6$ (even). Middle pair: 5 and 8.
- ▶ **Median = $(5 + 8)/2 = 6.5$**

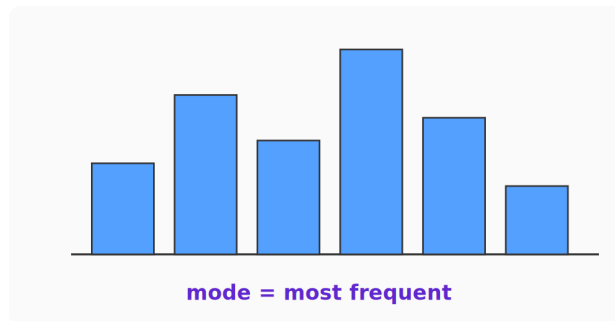
Average of the two middle values.

EXERCISES (EASY → DIFFICULT)

1. Find the median of 3, 5, 7.
2. Find the median of 2, 4, 6, 8.
3. Find the median of 1, 3, 3, 6, 9.
4. Find the median of 10, 20, 30, 40.
5. Find the median of 7, 2, 9, 4, 5.
6. Test marks 40, 55, 60, 65, 90 — find the median.
7. Find the median of 12, 14, 16, 18, 20, 22.
8. Why use the median instead of the mean for skewed data?
9. Find the median of 100, 5, 8, 6, 7 (note the outlier).
10. Six daily sales: R200, 250, 300, 350, 400, 1000. Find the median.

10.2 Mode

Mode (most frequent)



DEFINITIONS

Mode	The value that appears MOST OFTEN in the data set.
Bimodal / multimodal	A data set with two or more values tied for the highest frequency.
No mode	When every value appears the same number of times.



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KEY FORMULAS

- **Step 1:** Count how many times each value appears
- **Step 2:** The mode is the value with the HIGHEST count
- **Mode is the only average that works for non-numeric data** (e.g. favourite colour)

WORKED EXAMPLES

Example 1: Find the mode of 4, 2, 7, 4, 9, 4, 1.

- Count: 4 appears 3 times, all others once.
- **Mode = 4**
Most frequent value.

Example 2: Find the mode of 1, 2, 2, 3, 3, 4.

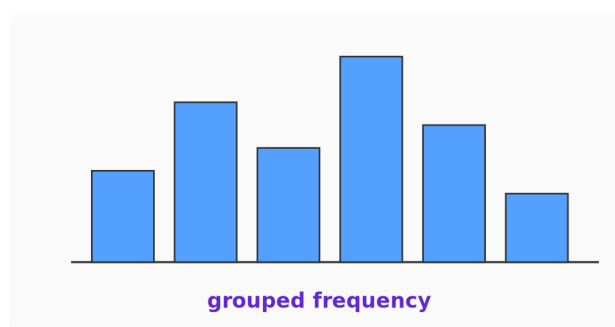
- 2 appears twice, 3 appears twice — tied.
- **Bimodal: 2 and 3**

EXERCISES (EASY → DIFFICULT)

1. Find the mode of 2, 3, 3, 5.
2. Find the mode of 4, 4, 6, 6, 6, 9.
3. Find the mode of 1, 2, 3, 4 (each once).
4. Find the mode of 5, 5, 7, 7, 9.
5. Shoe sizes sold: 6, 7, 7, 8, 7. Find the mode.
6. Which average is best for "most common size"?
7. Find the mode of 10, 12, 12, 12, 15, 15.
8. A shop records colours sold: red, red, blue, red, green. Modal colour?
9. Find the mode of 8, 9, 8, 9, 8.
10. Why might a data set have no mode?

10.3 Grouping data (frequency)

Grouping data (frequency tables)



DEFINITIONS

Frequency	How many times a value or class appears in a data set.
Class interval	A range of values, e.g. $0 \leq x < 10$. Used when data has many distinct values.
Frequency table	A table showing each class and its frequency, used to organise large data sets.



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KEY FORMULAS

- **Total frequency = total number of data items**
- **Mid-point of class:** (lower bound + upper bound) $\div$ 2
- **Estimated mean (grouped):** $\Sigma(\text{midpoint} \times \text{frequency}) \div \text{total frequency}$

WORKED EXAMPLES

Example 1: Marks (out of 20): 5, 8, 12, 15, 17, 7, 11, 13, 9, 18. Group into intervals of 5.

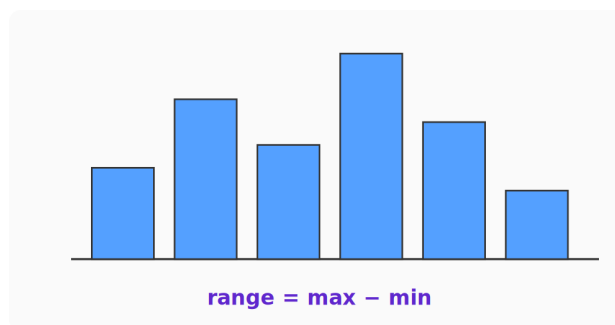
- ▶ 1–5: 1 value (5)
- ▶ 6–10: 3 values (8, 7, 9)
- ▶ 11–15: 4 values (12, 15, 11, 13)
- ▶ 16–20: 2 values (17, 18)

Total frequency = 10

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. In a frequency table, frequency means...
2. Marks 0–9:3, 10–19:5, 20–29:7. How many learners in total?
3. Which class has the highest frequency: [0–9]:3, [10–19]:5, [20–29]:7?
4. Tally: |||| || represents what frequency?
5. Group these into 0–4, 5–9: data 2, 5, 7, 3, 8. Give the two frequencies.
6. A survey of 30 people groups ages 0–19:10, 20–39:12, 40–59:8. How many are under 40?
7. From the age data, how many are 40 or older?
8. Why group data into classes?
9. Frequencies 4, 6, 10, 5 — what is the modal class (third class is widest count)?
10. A frequency table totals 50; classes are 12, 18, x, 6. Find x.

10.4 Range



DEFINITIONS

Range

A measure of spread: largest value minus smallest value.

KEY FORMULAS

- **Range = max – min**
- **Tells you the total spread of the data — but is sensitive to outliers**



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WORKED EXAMPLES

Example 1: Find the range of 4, 9, 2, 11, 7.

- ▶ Max = 11, Min = 2
- ▶ Range = $11 - 2 = 9$

Example 2: Range of test marks 45, 67, 50, 89, 72, 38.

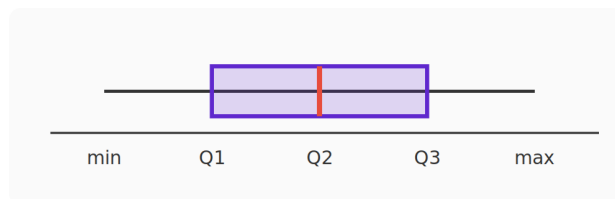
- ▶ Max = 89, Min = 38
- ▶ Range = 51

EXERCISES (EASY → DIFFICULT)

1. Find the range of 4, 9, 2, 15.
2. Find the range of 20, 35, 50.
3. Find the range of 7, 7, 7.
4. Find the range of 100, 250, 175, 90.
5. Range tells us the... of the data.
6. Daily temperatures 12° , 18° , 25° , 9° . Find the range.
7. Test marks 45, 60, 88, 90, 30. Find the range.
8. Which has a larger range: {4,5,6} or {1,5,9}?
9. Monthly sales (R1000s): 12, 20, 8, 15. Find the range.
10. Why is the range a weak measure of spread?

10.4 Inter-quartile range

Inter-quartile range (IQR)



DEFINITIONS

Quartiles	The three values (Q_1 , Q_2 , Q_3) that split sorted data into four equal-sized groups.
Q_1 (lower quartile)	The median of the LOWER half of the data — 25% of values are below it.
Q_3 (upper quartile)	The median of the UPPER half — 75% of values are below it.
Inter-quartile range	$IQR = Q_3 - Q_1$. The range of the middle 50% of the data — resists outliers.

KEY FORMULAS

- $IQR = Q_3 - Q_1$
- **Step 1:** Sort the data
- **Step 2:** Find Q_2 (the median)
- **Step 3:** Q_1 = median of values BELOW Q_2 ; Q_3 = median of values ABOVE Q_2



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WORKED EXAMPLES

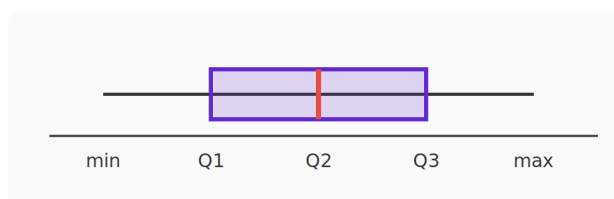
Example 1: Find the IQR of 2, 4, 5, 7, 8, 10, 12, 15, 18.

- ▶ Already sorted. $n = 9$.
- ▶ Median $Q_2 = 8$ (5th value).
- ▶ Lower half: 2, 4, 5, 7. $Q_1 = (4+5)/2 = 4.5$
- ▶ Upper half: 10, 12, 15, 18. $Q_3 = (12+15)/2 = 13.5$
- ▶ **IQR = $13.5 - 4.5 = 9$**

EXERCISES (EASY → DIFFICULT)

1. $Q_1 = 10$, $Q_3 = 25$. Find the IQR.
2. Find the IQR of 2, 4, 6, 8, 10, 12, 14, 16.
3. IQR measures the spread of the middle... of data.
4. $Q_1 = 30$, $Q_3 = 70$. Find the IQR.
5. Find Q_1 of 1, 2, 3, 4, 5, 6, 7, 8.
6. Test marks: 40, 55, 60, 65, 70, 75, 90. Find the IQR.
7. Why is IQR better than range for spread?
8. Find the IQR of 5, 8, 10, 12, 15, 18, 20, 25.
9. A box-plot has $Q_1 = 20$ and $Q_3 = 50$. State the IQR.
10. Data 3, 7, 8, 5, 12, 14, 21, 13, 18 — find the IQR (9 values).

10.5 Five-number summary



DEFINITIONS

Five-number summary

The five numbers that fully describe a data set's shape: minimum, Q_1 , median (Q_2), Q_3 , maximum.

Box-and-whisker plot

A visual diagram of the five-number summary — the box runs from Q_1 to Q_3 with a line at the median.

KEY FORMULAS

- **Five-number summary:** Min, Q_1 , Median, Q_3 , Max
- **Box plot:** box from Q_1 to Q_3 (containing 50% of data), whiskers to min and max
- **IQR = $Q_3 - Q_1$**

WORKED EXAMPLES



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**Example 1: Five-number summary of 5, 7, 8, 10, 12, 14, 17.**

- ▶ Sorted (already). $n = 7$.
- ▶ Min = 5, Max = 17
- ▶ Median = 10 (4th value)
- ▶ $Q_1 = 7$ (median of 5, 7, 8); $Q_3 = 14$ (median of 12, 14, 17)
- ▶ **5, 7, 10, 14, 17**

EXERCISES (EASY → DIFFICULT)

1. Name the five numbers in a five-number summary.
2. Find the five-number summary of 3, 7, 8, 12, 15.
3. A box-and-whisker plot displays the...
4. Marks 40, 55, 60, 65, 70, 75, 90 — give the 5-number summary.
5. From the marks above, find the IQR.
6. Daily sales (R): 200, 350, 400, 450, 600, 700 — find the median.
7. Find the range from the marks 40...90 above.
8. A box-plot: min 12, Q_1 20, med 28, Q_3 35, max 50. Find the IQR and range.
9. Five-number summary of 10, 12, 14, 16, 18, 20, 22, 24.
10. Which part of the box-plot shows the median?

End of Chapter 10 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Find the mean of 4, 6, 8.
2. Find the median of 2, 4, 6, 8.
3. Find the mode of 1, 2, 3, 4 (each once).
4. Tally: |||| || represents what frequency?
5. Range tells us the... of the data.
6. Test marks: 40, 55, 60, 65, 70, 75, 90. Find the IQR.
7. Find the range from the marks 40...90 above.
8. Five learners scored 8, 6, 7, 9, 10. Find the mean.
9. Find the median of 100, 5, 8, 6, 7 (note the outlier).
10. Why might a data set have no mode?



CHAPTER 12

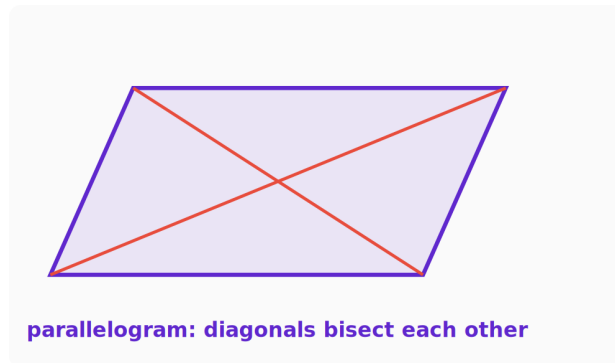
Euclidean geometry: proofs

12.1 Diagonals of a parallelogram

12.1 Diagonals of a rhombus

12.1 Conjecture about parallelogram diagonals

Diagonals of a parallelogram



DEFINITIONS

Parallelogram A quadrilateral with both pairs of opposite sides parallel.

Diagonal A line joining two non-adjacent vertices.

Bisect To cut into two EQUAL halves.

KEY FORMULAS

- **Conjecture:** the diagonals of a parallelogram **BISECT** each other
- **That is:** the point where they cross is the mid-point of BOTH diagonals
- **But the diagonals are NOT necessarily equal** (only in a rectangle)

WORKED EXAMPLES

Example 1: ABCD is a parallelogram with diagonals meeting at M. AM = 5 cm. Find AC.

- M is the midpoint of AC, so $MC = AM = 5$ cm.
- $AC = 5 + 5 = 10$ cm



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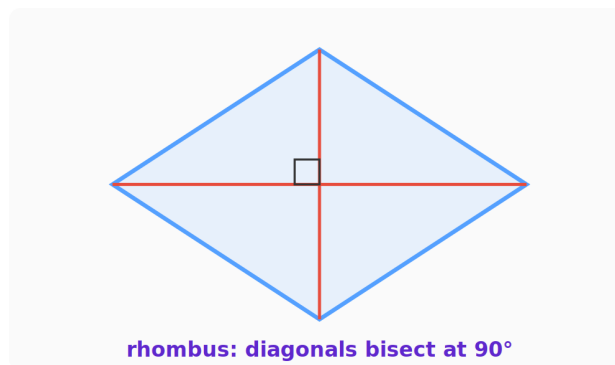
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EXERCISES (EASY → DIFFICULT)

1. What do the diagonals of a parallelogram do to each other?
2. In parallelogram ABCD, diagonals meet at M. If $AM = 5$, find MC.
3. If diagonal $BD = 12$, find each half (BM and MD).
4. Do the diagonals of a parallelogram have to be equal?
5. A diagonal of a parallelogram divides it into two... triangles.
6. If $AM = 2x$ and $MC = x + 4$ (diagonals bisect), find x .
7. Diagonals $AC = 10$ and $BD = 8$ meet at M. Find AM and BM.
8. Why are the two triangles formed by a diagonal congruent?
9. If diagonals bisect each other, the quadrilateral is a...
10. $BM = 3x - 1$ and $MD = x + 5$ (bisected). Find BM.

12.1 Conjecture about rhombus diagonals

Diagonals of a rhombus



DEFINITIONS

Rhombus A parallelogram with all FOUR sides equal in length.

Perpendicular bisectors Two lines that cross at 90° AND each cuts the other in half.

KEY FORMULAS

- **Conjecture (rhombus diagonals):**
 - bisect each other (parallelogram property)
 - are PERPENDICULAR (cross at 90°)
 - bisect the angles of the rhombus at each vertex

WORKED EXAMPLES

Example 1: ABCD is a rhombus. Diagonals AC and BD cross at M with $\angle AMB$. What is $\angle AMB$?

- Diagonals of a rhombus are perpendicular.
- $\angle AMB = 90^\circ$



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EXERCISES (EASY → DIFFICULT)

1. What is special about a rhombus's diagonals?
2. Do rhombus diagonals bisect the angles?
3. Rhombus diagonals meet at what angle?
4. A rhombus has diagonals 6 and 8. Find its side (Pythagoras on half-diagonals).
5. Is every rhombus a parallelogram?
6. Diagonals of a rhombus are 10 and 24. Find the side.
7. A rhombus diagonal bisects an 80° angle into...
8. Find the area of a rhombus with diagonals 6 and 8 ($\frac{1}{2} \cdot d_1 \cdot d_2$).
9. Why are a rhombus's diagonals perpendicular?
10. A rhombus has side 13 and one diagonal 10. Find the other diagonal.

End of Chapter 12 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. What do the diagonals of a parallelogram do to each other?
2. Do rhombus diagonals bisect the angles?
3. If diagonal $BD = 12$, find each half (BM and MD).
4. A rhombus has diagonals 6 and 8. Find its side (Pythagoras on half-diagonals).
5. A diagonal of a parallelogram divides it into two... triangles.
6. Diagonals of a rhombus are 10 and 24. Find the side.
7. Diagonals $AC = 10$ and $BD = 8$ meet at M. Find AM and BM.
8. Find the area of a rhombus with diagonals 6 and 8 ($\frac{1}{2} \cdot d_1 \cdot d_2$).
9. If diagonals bisect each other, the quadrilateral is a...
10. A rhombus has side 13 and one diagonal 10. Find the other diagonal.





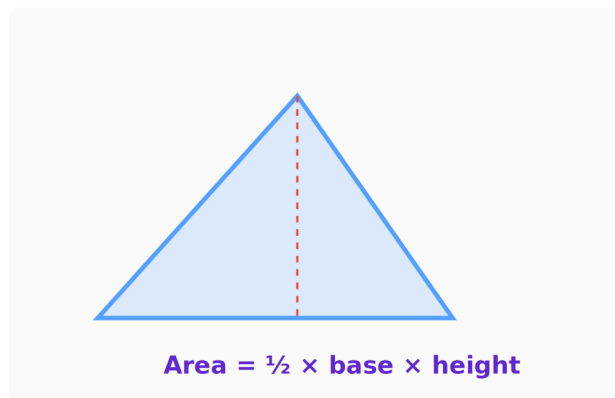
CHAPTER 13

Measurement

- 13.1** Area of a triangle
- 13.1** Area of a trapezium
- 13.1** Area of a circle
- 13.2** Volume of a cylinder
- 13.3** Volume of a sphere
- 13.3** Volume of a cone
- 13.4** Effect of multiplying dimensions by k

13.1 Area of triangle

Area of a triangle



DEFINITIONS

Base	Any one side of the triangle that you choose as the base.
Perpendicular height	The vertical distance from the chosen base to the opposite vertex (NOT the slant side).

KEY FORMULAS

- **Area of a triangle:** $A = \frac{1}{2} \times \text{base} \times \text{height}$
- **Height MUST be perpendicular** to the chosen base
- **Units:** if base and height are in cm, area is in cm^2



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WORKED EXAMPLES

Example 1: Triangle with base 12 cm, height 8 cm. Find the area.

$$\triangleright A = \frac{1}{2} \times 12 \times 8$$

$$\triangleright A = 48 \text{ cm}^2$$

Example 2: Triangle, base = 5 m, area = 25 m². Find the height.

$$\triangleright 25 = \frac{1}{2} \times 5 \times h$$

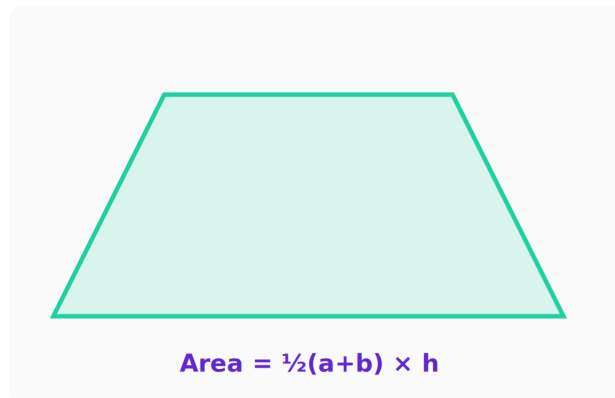
$$\triangleright h = 25 \times 2 / 5 = 10$$

$$\triangleright h = 10 \text{ m}$$

EXERCISES (EASY → DIFFICULT)

1. Find the area of a triangle with base 10 and height 6.
2. Find the area of a triangle with base 8 and height 5.
3. Find the area of a triangle with base 12 and height 7.
4. A triangle has area 24 and base 8. Find its height.
5. Area of a triangle formula?
6. A triangular sail has base 4 m and height 6 m. Find its area.
7. A triangular plot has base 30 m and height 20 m. Find the area.
8. A triangle has area 50 cm² and height 10 cm. Find the base.
9. A right triangle has legs 9 and 12 (as base and height). Find the area.
10. A roof gable triangle has base 6 m and height 2,5 m. Find its area.

13.1 Area of trapezium

Area of a trapezium

DEFINITIONS

Trapezium

A quadrilateral with one pair of parallel sides (the parallel sides are called the parallel "bases", a and b).

Perpendicular height

The shortest distance between the two parallel sides.



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KEY FORMULAS

- **Area of a trapezium:** $A = \frac{1}{2} \times (a + b) \times h$
- **where:** a, b = lengths of the two PARALLEL sides; h = perpendicular height between them

WORKED EXAMPLES

Example 1: A trapezium has parallel sides 6 cm and 10 cm, height 4 cm.

- ▶ $A = \frac{1}{2} \times (6 + 10) \times 4 = \frac{1}{2} \times 16 \times 4$
- ▶ $A = 32 \text{ cm}^2$

Example 2: Parallel sides 8 m and 14 m, height 5 m.

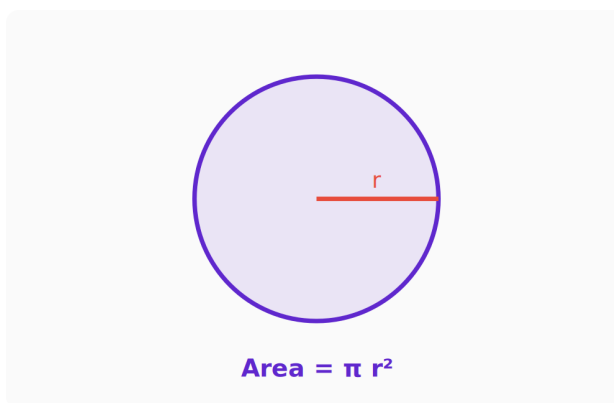
- ▶ $A = \frac{1}{2} \times (8 + 14) \times 5 = \frac{1}{2} \times 22 \times 5$
- ▶ $A = 55 \text{ m}^2$

EXERCISES (EASY → DIFFICULT)

1. Find the area of a trapezium with parallel sides 6 and 10, height 4.
2. Find the area with parallel sides 5 and 9, height 6.
3. Trapezium area formula?
4. Parallel sides 8 and 12, height 5. Find the area.
5. A trapezium has area 40, parallel sides 6 and 10. Find the height.
6. A trapezoidal field has parallel sides 30 m and 50 m, width 20 m. Find the area.
7. Parallel sides 7 and 11, height 8. Find the area.
8. A trapezium window has parallel sides 1,2 m and 0,8 m, height 0,5 m. Find the area.
9. Area 96, height 8, one parallel side 10. Find the other.
10. Parallel sides 15 and 25 cm, height 10 cm. Find the area.

13.1 Area of circle ($\pi = 3.14$)

Area of a circle



DEFINITIONS

Radius (r)	Distance from the centre of a circle to any point on the edge.
Diameter (d)	A chord through the centre — twice the radius. $d = 2r$.
π (pi)	The ratio of any circle's circumference to its diameter, approximately 3.14.



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KEY FORMULAS

- **Area of a circle:** $A = \pi \times r^2$
- **Circumference:** $C = 2\pi r$ (or πd)
- **Use $\pi \approx 3.14$** in this chapter

WORKED EXAMPLES

Example 1: Find the area of a circle with radius 7 cm. ($\pi = 3.14$)

- ▶ $A = 3.14 \times 7^2$
- ▶ $= 3.14 \times 49$
- ▶ **$A = 153.86 \text{ cm}^2$**

Example 2: A circle has diameter 20 m. Find its area.

- ▶ $r = 20 \div 2 = 10 \text{ m}$
Halve the diameter.
- ▶ $A = 3.14 \times 10^2 = 3.14 \times 100$
- ▶ **$A = 314 \text{ m}^2$**

EXERCISES (EASY → DIFFICULT)

1. Find the area of a circle of radius 10 ($\pi = 3.14$).
2. Find the area of a circle of radius 5 ($\pi = 3.14$).
3. Circle area formula?
4. Find the area of a circle of radius 2 ($\pi = 3.14$).
5. A circle has diameter 20 ($\pi = 3.14$). Find the area.
6. A circular tablecloth has radius 0,7 m ($\pi = 3.14$). Find the area.
7. A round pizza has radius 15 cm ($\pi = 3.14$). Find its area.
8. A circle has area 28.26 ($\pi = 3.14$). Find the radius.
9. A circular pond has radius 4 m ($\pi = 3.14$). Find its area.
10. A circular garden of radius 7 m ($\pi = 3.14$) is to be turfed. Find the area.



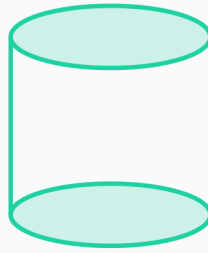
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13.2 Volume of cylinder ($\pi = 3.14$)

Volume of a cylinder



$$V = \pi r^2 h$$

DEFINITIONS

Cylinder	A solid with two parallel circular ends connected by a curved surface.
Volume	The space inside a 3-D solid, measured in cubic units (cm^3 , m^3).

KEY FORMULAS

- **Volume of a cylinder:** $V = \pi \times r^2 \times h$
- **That is:** area of circular base $\times$ height
- **Surface area:** $2\pi r^2 + 2\pi rh$ (top + bottom + curved side)

WORKED EXAMPLES

Example 1: Cylinder with $r = 5$ cm, $h = 10$ cm. Find V . ($\pi = 3.14$)

- $V = 3.14 \times 5^2 \times 10 = 3.14 \times 25 \times 10$
- **$V = 785 \text{ cm}^3$**

Example 2: Cylinder with $r = 4$ m, $h = 7$ m.

- $V = 3.14 \times 16 \times 7$
- **$V = 351.68 \text{ m}^3$**

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. Find the volume of a cylinder $r = 10$, $h = 5$ ($\pi = 3.14$).
2. Find the volume $r = 2$, $h = 10$ ($\pi = 3.14$).
3. Cylinder volume formula?
4. Find the volume $r = 5$, $h = 4$ ($\pi = 3.14$).
5. A cylinder $r = 3$, $h = 7$ ($\pi = 3.14$). Find V .
6. A water tank is a cylinder $r = 1$ m, $h = 2$ m ($\pi = 3.14$). Find V in m^3 .
7. A tin $r = 7$ cm, $h = 10$ cm ($\pi = 3.14$). Find the volume.
8. A cylinder has volume 1 256 and $r = 10$ ($\pi = 3.14$). Find the height.
9. A cylindrical drum $r = 0,5$ m, $h = 1$ m ($\pi = 3.14$). Find V in litres ($1 \text{ m}^3 = 1000 \square$).
10. A pipe $r = 2$ cm, length 100 cm ($\pi = 3.14$). Find the volume.

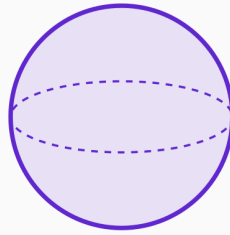


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13.3 Volume of sphere

Volume of a sphere



$$V = \frac{4}{3} \pi r^3$$

DEFINITIONS

Sphere

A perfectly round 3-D shape — every point on its surface is the same distance r from the centre.

KEY FORMULAS

- **Volume of a sphere:** $V = \frac{4}{3} \times \pi \times r^3$
- **Surface area:** $SA = 4\pi r^2$

WORKED EXAMPLES

Example 1: Sphere with $r = 3$ cm. Find V . ($\pi = 3.14$)

- ▶ $V = \frac{4}{3} \times 3.14 \times 3^3$
- ▶ $= \frac{4}{3} \times 3.14 \times 27$
- ▶ $V \approx 113.04 \text{ cm}^3$

Example 2: Sphere with $r = 6$ m.

- ▶ $V = \frac{4}{3} \times 3.14 \times 216$
- $6^3 = 216$
- ▶ $V \approx 904.32 \text{ m}^3$

EXERCISES (EASY → DIFFICULT)

1. Find the volume of a sphere $r = 3$ ($\pi = 3.14$).
2. Sphere volume formula?
3. Find the volume $r = 6$ ($\pi = 3.14$).
4. Find the volume $r = 1$ ($\pi = 3.14$).
5. Find the volume $r = 2$ ($\pi = 3.14$).
6. A soccer ball $r = 11$ cm ($\pi = 3.14$). Find the volume.
7. A spherical tank $r = 0,5$ m ($\pi = 3.14$). Find V in m^3 .
8. A marble has $r = 1$ cm ($\pi = 3.14$). Find its volume.
9. A sphere has volume 33.49 ($\pi = 3.14$). Find r .
10. A globe has radius 20 cm ($\pi = 3.14$). Find the volume.

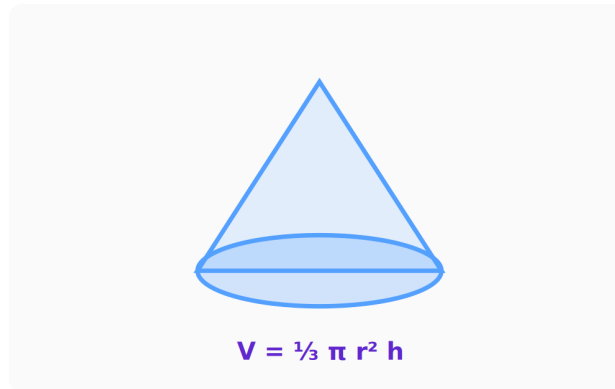


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13.3 Volume of cone

Volume of a cone



DEFINITIONS

- | | |
|---|--|
| Cone | A solid with a circular base tapering to a single point (the apex). |
| Slant height (ℓ) | The distance from the apex to a point on the edge of the base — useful for surface area. |

KEY FORMULAS

- **Volume of a cone:** $V = (1/3) \times \pi \times r^2 \times h$
- **That is:** ONE-THIRD of the cylinder with the same base and height
- **Surface area:** $\pi r^2 + \pi r \ell$

WORKED EXAMPLES

Example 1: Cone with $r = 6$ cm, $h = 9$ cm. Find V . ($\pi = 3.14$)

- ▶ $V = (1/3) \times 3.14 \times 36 \times 9$
- ▶ $V = 339.12 \text{ cm}^3$

Example 2: Cone with $r = 5$ m, $h = 12$ m.

- ▶ $V = (1/3) \times 3.14 \times 25 \times 12$
- ▶ $V = 314 \text{ m}^3$

EXERCISES (EASY → DIFFICULT)

- Find the volume of a cone $r = 3$, $h = 6$ ($\pi = 3.14$).
- Cone volume formula?
- Find the volume $r = 5$, $h = 9$ ($\pi = 3.14$).
- Find the volume $r = 2$, $h = 12$ ($\pi = 3.14$).
- Find the volume $r = 6$, $h = 10$ ($\pi = 3.14$).
- An ice-cream cone $r = 3$ cm, $h = 12$ cm ($\pi = 3.14$). Find the volume.
- A conical pile of sand $r = 2$ m, $h = 1.5$ m ($\pi = 3.14$). Find V .
- A cone has volume 100.48 and $r = 4$ ($\pi = 3.14$). Find the height.
- A party hat is a cone $r = 7$, $h = 24$ ($\pi = 3.14$). Find the volume.
- A cone and cylinder share r and h . The cone's volume is what fraction of the cylinder's?

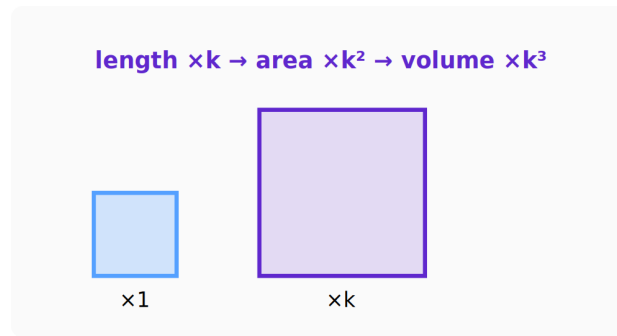


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13.4 Effect of multiplying dimensions by k



DEFINITIONS

Scale factor (k) A multiplier applied to ALL the linear dimensions of a shape.

Linear, area, volume scaling Length scales by k , area scales by k^2 , volume scales by k^3 .

KEY FORMULAS

- **Length $\times k$:** all lengths multiplied by k
- **Area $\times k^2$:** areas (and surface areas) multiply by k^2
- **Volume $\times k^3$:** volumes multiply by k^3
- **Memory:** the exponent on k = the dimension being measured

WORKED EXAMPLES

Example 1: A square has area 16 cm^2 . Scale all sides by 3. What is the new area?

- $k = 3$, so area $\times k^2 = 9$
- New area = $16 \times 9 = 144 \text{ cm}^2$

Example 2: A cube has volume 27 cm^3 . Scale all sides by 4. What's the new volume?

- $k = 4$, $k^3 = 64$
- New volume = $27 \times 64 = 1\,728 \text{ cm}^3$

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. A square's side is doubled. The area becomes...
2. A cube's side is doubled. The volume becomes...
3. A length is tripled. The area becomes...
4. A length is tripled. The volume becomes...
5. A length is halved. The volume becomes...
6. A cylinder's r and h are both doubled. Volume $\times$?
7. A 1:10 model — the real surface area is $\times$?
8. All dimensions $\times 1.5$. Volume $\times$?
9. A box of volume 30 cm^3 has every dimension $\times k$, giving 240 cm^3 . Find k .
10. A photo of area 12 cm^2 has each side $\times 4$. New area?



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End of Chapter 13 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Find the area of a triangle with base 10 and height 6.
2. Find the area with parallel sides 5 and 9, height 6.
3. Circle area formula?
4. Find the volume $r = 5$, $h = 4$ ($\pi = 3.14$).
5. Find the volume $r = 2$ ($\pi = 3.14$).
6. An ice-cream cone $r = 3$ cm, $h = 12$ cm ($\pi = 3.14$). Find the volume.
7. A 1:10 model — the real surface area is $\times$?
8. A triangle has area 50 cm^2 and height 10 cm. Find the base.
9. Area 96, height 8, one parallel side 10. Find the other.
10. A circular garden of radius 7 m ($\pi = 3.14$) is to be turfed. Find the area.



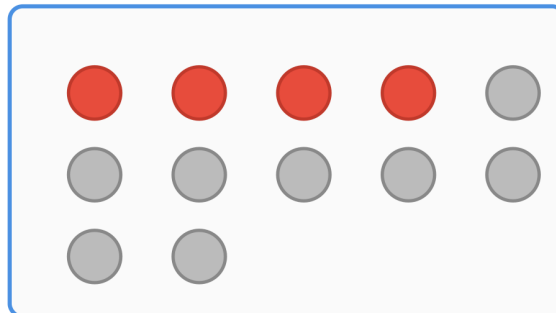
CHAPTER 14

Probability

- 14.1** Theoretical probability
- 14.2** Relative frequency (experimental probability)
- 14.3** Venn diagram (2 sets)
- 14.4** Union and intersection
- 14.5** Probability identities
- 14.6** Mutually exclusive events
- 14.7** Complementary events

14.1 Theoretical probability

Sample space S (12 outcomes)



$$P(A) = 4 \div 12 = 1/3$$

DEFINITIONS

Outcome	A single possible result of an experiment (e.g. rolling a 4 on a die).
Sample space (S)	The set of ALL possible outcomes.
Event (E)	A subset of the sample space — the outcomes you are interested in.
Probability	A number between 0 (impossible) and 1 (certain) describing how likely an event is.



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KEY FORMULAS

- **Theoretical probability:** $P(E) = n(E) / n(S)$
- **Where:** $n(E)$ = number of favourable outcomes; $n(S)$ = total number of outcomes
- **Range:** $0 \leq P(E) \leq 1$ (also written as a percentage 0%-100%)

WORKED EXAMPLES

Example 1: Roll a fair die. What is $P(\text{rolling a 4})$?

- ▶ $n(S) = 6$ (six possible faces)
- ▶ $n(E) = 1$ (only one way to get a 4)
- ▶ $P = 1/6$

Example 2: A bag has 5 red and 3 blue marbles. Find $P(\text{blue})$.

- ▶ $n(S) = 5 + 3 = 8$
Total marbles.
- ▶ $n(E) = 3$ (blue marbles)
- ▶ $P(\text{blue}) = 3/8$

EXERCISES (EASY → DIFFICULT)

1. A fair coin is tossed. Find $P(\text{heads})$.
2. A die is rolled. Find $P(\text{a 4})$.
3. A die is rolled. Find $P(\text{an even number})$.
4. A bag has 3 red and 2 blue. Find $P(\text{red})$.
5. Probability formula?
6. A die is rolled. Find $P(\text{a number greater than 4})$.
7. A card is drawn from 52. Find $P(\text{an ace})$.
8. A spinner has 8 equal parts, 3 red. Find $P(\text{red})$.
9. A bag has 5 red, 3 green, 2 blue. Find $P(\text{green})$.
10. In a class of 30, 18 are girls. Find $P(\text{a randomly chosen learner is a boy})$.

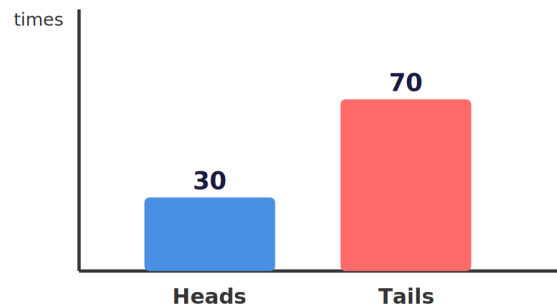


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14.2 Relative frequency

Relative frequency (experimental probability)



Heads: $30/100$ · Tails: $70/100$

DEFINITIONS

Relative frequency

The PROPORTION of times an event happens in repeated trials: number of successes $\div$ total trials.

Law of large numbers

As the number of trials grows, the relative frequency gets CLOSER to the theoretical probability.

KEY FORMULAS

- **Relative frequency:** $RF = (\text{number of successes}) \div (\text{total trials})$
- This is also called **experimental probability**
- $RF \rightarrow P(E)$ as the number of trials $\rightarrow \infty$

WORKED EXAMPLES

Example 1: A coin is tossed 200 times and lands heads 92 times. Find the relative frequency.

- ▶ $RF = 92 \div 200$
- ▶ **$RF = 0.46$**

Close to but not exactly 0.5.

Example 2: A drawing pin is tossed 50 times, landing point-up 18 times. What's the experimental probability?

- ▶ $RF = 18 \div 50$
- ▶ **$RF = 0.36$**

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. A coin is tossed 100 times, landing heads 54 times. Find the relative frequency of heads.
2. Relative frequency = ...
3. A die rolled 60 times shows a six 12 times. Relative frequency of a six?
4. As trials increase, relative frequency approaches the...
5. A spinner spun 50 times lands on red 20 times. Relative frequency?
6. A factory tests 200 bulbs; 6 are faulty. Relative frequency of faulty?
7. Out of 500 shoppers, 350 buy bread. Estimate $P(\text{buys bread})$.



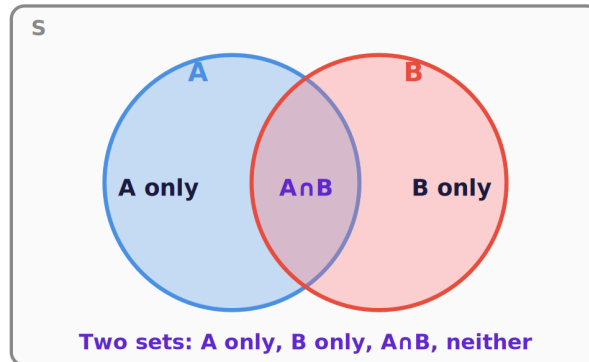
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8. A coin gives 540 heads in 1 000 tosses. Is it fair?
9. A team wins 18 of 24 games. Relative frequency of a win?
10. A survey of 80 finds 28 prefer tea. Estimate $P(\text{prefers tea})$.

14.3 Venn diagram (2 sets)



DEFINITIONS

Venn diagram	A picture using overlapping circles to show how two or more sets relate.
Universal set (S)	Everything we are considering — the rectangle around the circles.
A only	Inside circle A but NOT inside B.
$A \cap B$ (intersection)	The overlap — inside BOTH A and B.

KEY FORMULAS

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ (counted overlap only once)
- $n(\text{only } A) = n(A) - n(A \cap B)$
- $n(\text{neither}) = n(S) - n(A \cup B)$

WORKED EXAMPLES

Example 1: 30 students. 18 like maths (M), 12 like science (Sc), 8 like BOTH. How many like NEITHER?

► $n(M \cup \text{Sc}) = 18 + 12 - 8 = 22$

Inclusion-exclusion.

► $\text{Neither} = 30 - 22 = 8$

► **8 students like neither**

EXERCISES (EASY → DIFFICULT)

1. In a Venn diagram, the overlap of A and B represents...
2. $n(A) = 20$, $n(B) = 15$, $n(A \cap B) = 5$. Find $n(A \cup B)$.
3. Of 50, 30 like tea, 25 like coffee, 10 like both. How many like at least one?
4. From the above, how many like neither?
5. $n(\text{A only})$ when $n(A) = 18$, $n(A \cap B) = 7$.
6. In a class of 40, 22 do Maths, 18 do Science, 8 do both. How many do only Maths?

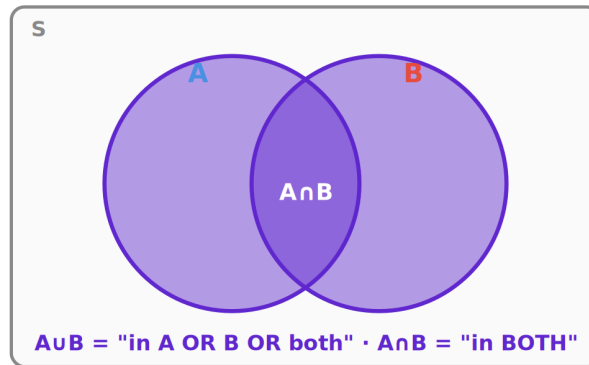


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7. From the same class, how many do neither?
8. Of 100, 60 buy bread, 45 buy milk, 25 both. Find $P(\text{bread or milk})$.
9. Of 80, 50 own a phone, 30 a tablet, 20 both. How many own exactly one?
10. A and B are mutually exclusive: $n(A)=12$, $n(B)=9$ of 30. Find $P(A \text{ or } B)$.

14.4 Union and intersection



DEFINITIONS

Union ($A \cup B$) All elements in A OR B (or both). The outline of both circles together.

Intersection ($A \cap B$) All elements in A AND B. The overlap region only.

KEY FORMULAS

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- If mutually exclusive (no overlap): $P(A \cup B) = P(A) + P(B)$

WORKED EXAMPLES

Example 1: $P(A) = 0.6$, $P(B) = 0.5$, $P(A \cap B) = 0.2$. Find $P(A \cup B)$.

- ▶ $P(A \cup B) = 0.6 + 0.5 - 0.2$
- ▶ $P(A \cup B) = 0.9$

EXERCISES (EASY → DIFFICULT)

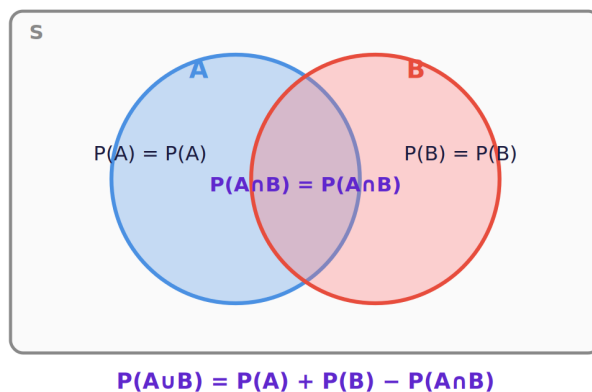
1. $A \cup B$ means...
2. $A \cap B$ means...
3. $A = \{1,2,3\}$, $B = \{3,4,5\}$. Find $A \cup B$.
4. $A = \{1,2,3\}$, $B = \{3,4,5\}$. Find $A \cap B$.
5. $n(A) = 10$, $n(B) = 8$, $n(A \cap B) = 3$. Find $n(A \cup B)$.
6. Of 100 learners, 40 play soccer, 30 netball, 10 both. Find $n(\text{soccer or netball})$.
7. From that, find $P(\text{plays neither})$.
8. If $A \cap B = \emptyset$, the events are...
9. $n(A \cup B) = 25$, $n(A) = 15$, $n(B) = 14$. Find $n(A \cap B)$.
10. $A = \text{even numbers}$, $B = \text{numbers } > 3 \text{ on a die}$. Find $A \cap B$.



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14.5 Probability identities



DEFINITIONS

Identity	A relationship between probabilities that is ALWAYS true, no matter the events.
Sample-space identity	The probabilities of all possible outcomes add to 1.

KEY FORMULAS

- $0 \leq P(E) \leq 1$ for every event E
- $P(S) = 1$ (certain that something in S happens)
- $P(\text{impossible}) = 0$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A') = 1 - P(A)$ (complement)

WORKED EXAMPLES

Example 1: $P(\text{rain}) = 0.3$. What is $P(\text{no rain})$?

- $P(\text{no rain}) = 1 - 0.3$
Complement rule.
- $P(\text{no rain}) = 0.7$

Example 2: $P(A) = 0.4$, $P(B) = 0.5$, A and B mutually exclusive. Find $P(A \cup B)$.

- Mutually exclusive $\implies P(A \cap B) = 0$
- $P(A \cup B) = 0.4 + 0.5 - 0 = 0.9$
- $P(A \cup B) = 0.9$

EXERCISES (EASY → DIFFICULT)

1. State the addition rule for probability.
2. $P(A) = 0.5$, $P(B) = 0.4$, $P(A \text{ and } B) = 0.2$. Find $P(A \text{ or } B)$.
3. $P(A) = 0.6$, $P(B) = 0.3$, mutually exclusive. Find $P(A \text{ or } B)$.
4. $P(A \text{ or } B) = 0.8$, $P(A) = 0.5$, $P(B) = 0.4$. Find $P(A \text{ and } B)$.
5. For mutually exclusive events, $P(A \text{ and } B) = \dots$
6. $P(\text{red}) = 0.3$, $P(\text{blue}) = 0.25$ (exclusive). Find $P(\text{red or blue})$.
7. $P(A) = 0.7$, $P(B) = 0.6$, $P(A \text{ or } B) = 0.9$. Find $P(A \text{ and } B)$.

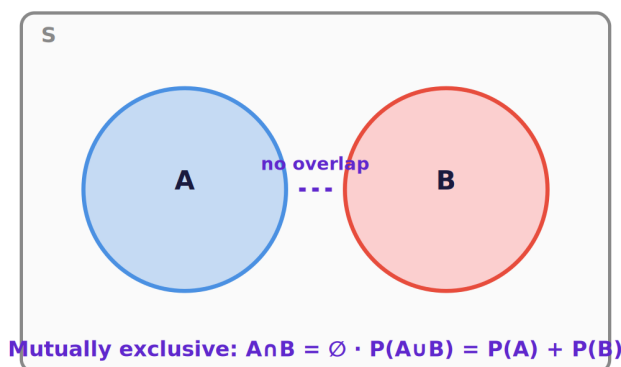


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8. Of 100, 60 like tea, 50 coffee, 30 both. Find $P(\text{tea or coffee})$.
9. $P(A) = 0.45$, $P(A \text{ and } B) = 0.15$, $P(A \text{ or } B) = 0.7$. Find $P(B)$.
10. Two exclusive events have $P(A \text{ or } B) = 1$. What does that mean?

14.6 Mutually exclusive events



DEFINITIONS

Mutually exclusive

Two events that CANNOT happen at the same time, e.g. rolling a 2 and rolling a 5 on the same die.

Empty intersection

For mutually exclusive events, $A \cap B = \emptyset$ and so $P(A \cap B) = 0$.

KEY FORMULAS

- If A and B are mutually exclusive:
- $P(A \cap B) = 0$
- $P(A \cup B) = P(A) + P(B)$
- **Test:** Can both events occur in a single trial? If NO $\rightarrow$ mutually exclusive.

WORKED EXAMPLES

Example 1: Roll a die. Are A = 'rolling a 2' and B = 'rolling a 5' mutually exclusive?

- Both can't happen on the SAME roll.
- **Yes — mutually exclusive**

$$P(A \cup B) = 1/6 + 1/6 = 1/3.$$

Example 2: Roll a die. A = 'even', B = 'less than 4'. Mutually exclusive?

- $A = \{2, 4, 6\}$, $B = \{1, 2, 3\}$
- Both contain 2, so they can happen together.
- **NOT mutually exclusive**

EXERCISES (EASY $\rightarrow$ DIFFICULT)

1. Mutually exclusive events cannot...
2. Rolling a 2 and a 5 on one die — mutually exclusive?
3. For mutually exclusive A, B: $P(A \text{ and } B) = \dots$
4. $P(A) = 0.3$, $P(B) = 0.5$ (exclusive). Find $P(A \text{ or } B)$.

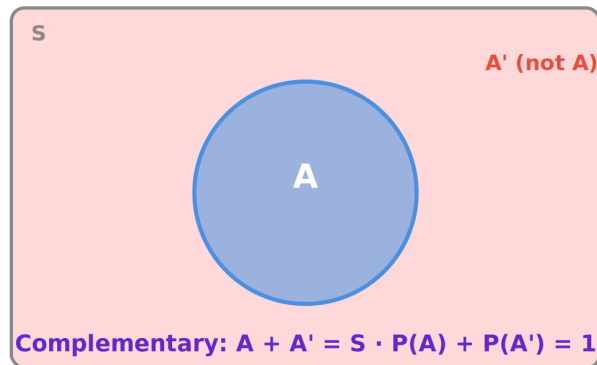


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5. Drawing a king and a queen in one draw — exclusive?
6. Are "even" and "odd" on a die mutually exclusive?
7. $P(A) = 0.4$, $P(B) = 0.35$ (exclusive). Find $P(\text{neither})$.
8. Are "red" and "face card" in a deck mutually exclusive?
9. Two exclusive events have P 0.45 and 0.4. Find $P(\text{one or the other})$.
10. If $P(A \text{ or } B) = P(A) + P(B)$, the events are...

14.7 Complementary events



DEFINITIONS

Complement (A') Everything in the sample space that is NOT in event A.

Complementary pair A and A' together cover the whole sample space and don't overlap.

KEY FORMULAS

- $P(A) + P(A') = 1$
- $P(A') = 1 - P(A)$
- Useful when 'NOT happening' is easier to count than 'happening'

WORKED EXAMPLES

Example 1: $P(A) = 0.35$. Find $P(A')$.

- $P(A') = 1 - 0.35$
- $P(A') = 0.65$

Example 2: A bag has 3 red, 7 blue marbles. Find $P(\text{NOT red})$.

- $P(\text{red}) = 3/10$
- $P(\text{not red}) = 1 - 3/10 = 7/10$
- $P(\text{not red}) = 7/10$



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**EXERCISES (EASY → DIFFICULT)**

1. $P(A) = 0.3$. Find $P(\text{not } A)$.
2. $P(\text{rain}) = 0.25$. Find $P(\text{no rain})$.
3. A die is rolled. Find $P(\text{not a } 6)$.
4. Complementary events sum to...
5. $P(A') = 0.55$. Find $P(A)$.
6. The weather app says 30% rain. Find $P(\text{stays dry})$.
7. $P(\text{passing}) = 0.8$. Of 200 learners, how many are expected to fail?
8. A machine is faulty 4% of the time. Find $P(\text{works})$.
9. A bag has 3 red, 7 blue. Find $P(\text{not red})$.
10. $P(\text{winning}) = 1/50$. Find $P(\text{not winning})$.

End of Chapter 14 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. A fair coin is tossed. Find $P(\text{heads})$.
2. Relative frequency = ...
3. Of 50, 30 like tea, 25 like coffee, 10 like both. How many like at least one?
4. $A = \{1,2,3\}$, $B = \{3,4,5\}$. Find $A \cap B$.
5. For mutually exclusive events, $P(A \text{ and } B) = \dots$
6. Are "even" and "odd" on a die mutually exclusive?
7. $P(\text{passing}) = 0.8$. Of 200 learners, how many are expected to fail?
8. A spinner has 8 equal parts, 3 red. Find $P(\text{red})$.
9. A team wins 18 of 24 games. Relative frequency of a win?
10. A and B are mutually exclusive: $n(A)=12$, $n(B)=9$ of 30. Find $P(A \text{ or } B)$.

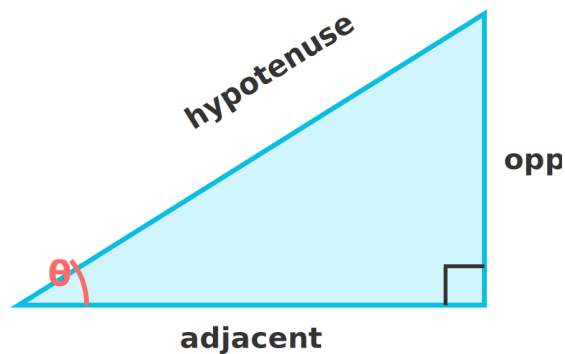


CHAPTER 10

Trigonometry

- 10.1** Defining the trigonometric ratios
- 10.2** Reciprocal ratios
- 10.3** Calculator skills
- 10.4** Special angles
- 10.5** Solving trigonometric equations
- 10.6** Ratios on the Cartesian plane

10.1 Defining the trigonometric ratios



DEFINITIONS

Hypotenuse	The LONGEST side of a right-angled triangle — opposite the right angle.
Opposite (to θ)	The side ACROSS from the angle θ .
Adjacent (to θ)	The side NEXT TO angle θ that is NOT the hypotenuse.
SOH CAH TOA	Memory aid: $\text{Sin} = \text{Opp}/\text{Hyp}$, $\text{Cos} = \text{Adj}/\text{Hyp}$, $\text{Tan} = \text{Opp}/\text{Adj}$.

EXERCISES (EASY → DIFFICULT)

- In a right triangle, $\sin \theta = ?$
- $\cos \theta = ?$
- $\tan \theta = ?$
- A right triangle has opposite 3, hypotenuse 5. Find $\sin \theta$.
- Opposite 4, adjacent 3. Find $\tan \theta$.
- Adjacent 8, hypotenuse 10. Find $\cos \theta$.

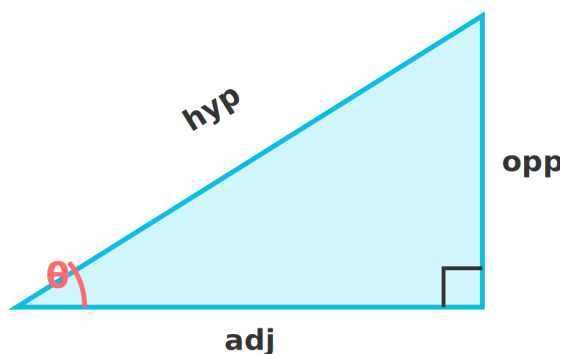


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7. If $\sin \theta = 3/5$ (acute), find $\cos \theta$.
8. A ladder makes opposite 6 m, hypotenuse 10 m. Find $\sin \theta$.
9. If $\tan \theta = 1$, what is θ ?
10. A 3-4-5 triangle: find $\sin \theta$, $\cos \theta$, $\tan \theta$ for the angle opposite the side 3.

10.2 Reciprocal ratios



DEFINITIONS

Reciprocal	The "flipped" version of a fraction — for a/b , the reciprocal is b/a .
cosec θ	$1 / \sin \theta$ — the reciprocal of sine.
sec θ	$1 / \cos \theta$ — the reciprocal of cosine.
cot θ	$1 / \tan \theta$ — the reciprocal of tangent.

EXERCISES (EASY → DIFFICULT)

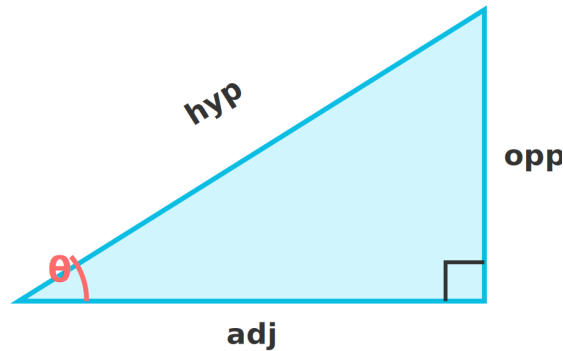
1. cosec θ is the reciprocal of...
2. sec θ is the reciprocal of...
3. cot θ is the reciprocal of...
4. If $\sin \theta = 1/2$, find cosec θ .
5. If $\cos \theta = 4/5$, find sec θ .
6. If $\tan \theta = 3/4$, find cot θ .
7. Write cosec θ in terms of sides.
8. If $\sin \theta = 0.5$, find cosec θ .
9. sec $\theta = 2$ means $\cos \theta = \dots$
10. If cot $\theta = 1$, find tan θ and θ .



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10.3 Calculator skills



DEFINITIONS

DEG mode	Calculator mode where angles are measured in degrees (0° – 360°). Always check this first!
RAD mode	Calculator mode using radians instead — DON'T use it for school trig at this level.

EXERCISES (EASY → DIFFICULT)

1. Evaluate $\sin 30^\circ$ (calculator).
2. Evaluate $\cos 60^\circ$.
3. Evaluate $\tan 45^\circ$.
4. Evaluate $\sin 90^\circ$.
5. Evaluate $\cos 0^\circ$.
6. Evaluate $\sin 53^\circ$ (2 dp).
7. Evaluate $\tan 30^\circ$ (2 dp).
8. Find θ if $\sin \theta = 0.5$ (acute).
9. Find θ if $\cos \theta = 0.5$ (acute).
10. Find θ if $\tan \theta = 1.732$ (2 dp).

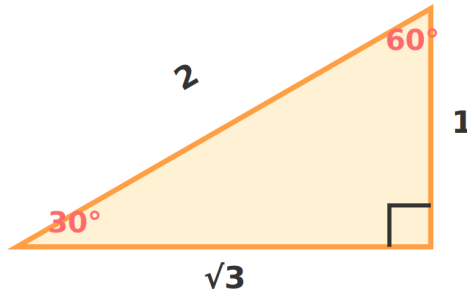


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10.4 Special angles



DEFINITIONS

Special angles	$30^\circ, 45^\circ, 60^\circ$ — angles whose sin, cos, tan are exact values you should memorise.
30-60-90 triangle	A right triangle with side ratios $1 : \sqrt{3} : 2$.
45-45-90 triangle	An isosceles right triangle with side ratios $1 : 1 : \sqrt{2}$.

EXERCISES (EASY → DIFFICULT)

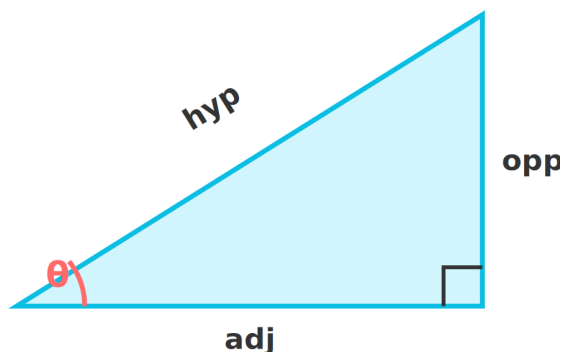
1. Give the exact value of $\sin 30^\circ$.
2. Give the exact value of $\cos 30^\circ$.
3. Give the exact value of $\tan 45^\circ$.
4. Give the exact value of $\sin 60^\circ$.
5. Give the exact value of $\cos 45^\circ$.
6. Give the exact value of $\tan 60^\circ$.
7. Give the exact value of $\sin 45^\circ$.
8. Give the exact value of $\tan 30^\circ$.
9. Evaluate $\sin 30^\circ + \cos 60^\circ$.
10. Evaluate $\sin^2 45^\circ + \cos^2 45^\circ$.



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10.5 Solving trigonometric equations



DEFINITIONS

Inverse trig function

$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ — given a ratio, return the angle.

Solving a trig equation

Isolate the trig function, then apply the inverse to find the angle.

EXERCISES (EASY → DIFFICULT)

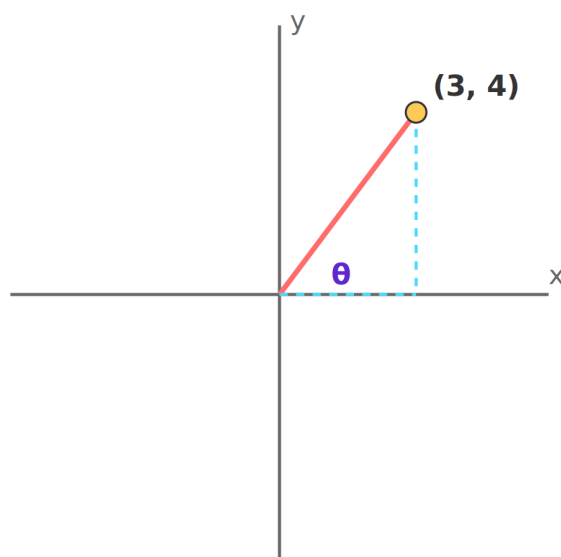
1. Solve $\sin \theta = 0.5$ for $0^\circ \leq \theta \leq 90^\circ$.
2. Solve $\cos \theta = 0.5$ for $0^\circ \leq \theta \leq 90^\circ$.
3. Solve $\tan \theta = 1$ for $0^\circ \leq \theta \leq 90^\circ$.
4. Solve $\sin \theta = 1$.
5. Solve $\cos \theta = 1$.
6. Solve $2 \sin \theta = 1$ (acute).
7. Solve $\tan \theta = \sqrt{3}$ (acute).
8. Solve $\sin \theta = 0.8$ (2 dp, acute).
9. Solve $3 \tan \theta = 3$ (acute).
10. Solve $\cos \theta = 0.866$ (acute, 0 dp).



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10.6 Ratios on the Cartesian plane



DEFINITIONS

Terminal arm The rotating arm of an angle on the Cartesian plane, starting from the positive x-axis.

$r = \sqrt{x^2 + y^2}$ The distance from the origin to a point (x, y) on the terminal arm.

EXERCISES (EASY → DIFFICULT)

1. In which quadrant are all ratios positive?
2. In Q2, which ratio is positive?
3. In Q3, which ratio is positive?
4. In Q4, which ratio is positive?
5. A point $(3, 4)$ — find r (distance to origin).
6. For point $(3, 4)$, find $\sin \theta$ ($= y/r$).
7. For point $(3, 4)$, find $\cos \theta$ ($= x/r$).
8. For point $(3, 4)$, find $\tan \theta$ ($= y/x$).
9. A point $(-3, 4)$ is in which quadrant, and is $\cos \theta$ $+/ -$?
10. Use CAST to state the sign of $\cos \theta$ in Q3.



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End of Chapter 10 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. In a right triangle, $\sin \theta = ?$
2. $\sec \theta$ is the reciprocal of...
3. Evaluate $\tan 45^\circ$.
4. Give the exact value of $\sin 60^\circ$.
5. Solve $\cos \theta = 1$.
6. For point (3, 4), find $\sin \theta (= y/r)$.
7. If $\sin \theta = 3/5$ (acute), find $\cos \theta$.
8. If $\sin \theta = 0.5$, find $\operatorname{cosec} \theta$.
9. Find θ if $\cos \theta = 0.5$ (acute).
10. Evaluate $\sin^2 45^\circ + \cos^2 45^\circ$.





CHAPTER 11

Functions

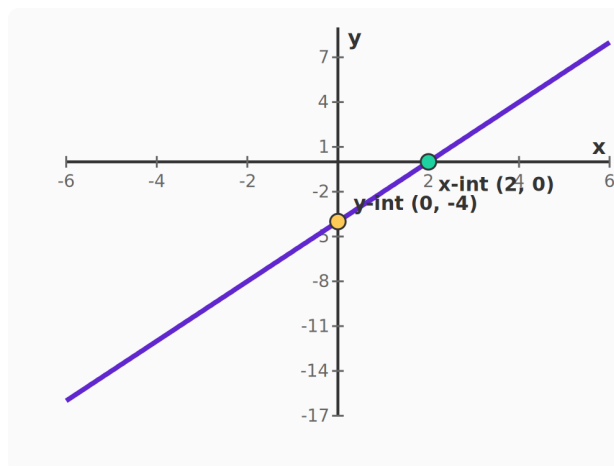
10.F1 Straight Line function

10.F2 Quadratic function

10.F3 Hyperbolic function

10.F4 Exponential function

10.F1 Straight Line function



DEFINITIONS

Linear function A function of the form $y = mx + c$ whose graph is a straight line.

Gradient (m) How steep the line is. Positive rises, negative falls.

y-intercept (c) Where the line crosses the y-axis — found by setting $x = 0$.

x-intercept Where the line crosses the x-axis — found by setting $y = 0$.

KEY FORMULAS



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- **Standard form:** $y = mx + c$
- **m** = gradient (slope) — the coefficient of x
- **c** = y -intercept — the constant term
- **Y-intercept:** set $x = 0$, so $y = c$
- **X-intercept:** set $y = 0$, so $x = -c/m$
- **Domain:** $x \in \mathbb{R}$
- **Range:** $y \in \mathbb{R}$

WORKED EXAMPLES

Example 1: For $y = 2x - 4$, find the y -intercept and x -intercept.

► **Y-intercept:** set $x = 0 \rightarrow y = 2(0) - 4 = -4$.

Y-intercept is $(0, -4)$.

► **X-intercept:** set $y = 0 \rightarrow 0 = 2x - 4 \rightarrow 2x = 4 \rightarrow x = 2$.

X-intercept is $(2, 0)$.

Example 2: For $y = -x + 3$, find the gradient and y -intercept.

► Compare to $y = mx + c$. **m** = -1 , **c** = 3 .

Read the coefficients directly.

► Gradient is -1 (line slopes down). Y -intercept is 3 .

Negative gradient means the line falls left to right.

Example 3: For $y = 3x - 9$, find both intercepts.

► **Y-intercept:** set $x = 0 \rightarrow y = -9$.

Constant term is the y -intercept.

► **X-intercept:** $0 = 3x - 9 \rightarrow x = 3$.

Solve for x .

EXERCISES (EASY $\rightarrow$ DIFFICULT)

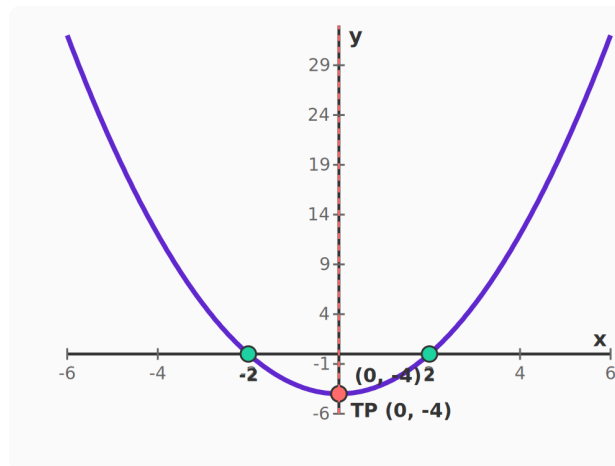
1. Find the gradient and y -intercept of $y = 2x + 3$.
2. Find the x -intercept of $y = 2x - 6$.
3. Find the equation of the line, gradient 3 through $(0, 1)$.
4. Where does $y = x$ meet $y = -x + 4$?
5. State the gradient of $y = -\frac{1}{2}x + 5$.
6. A taxi fare is $y = 8x + 12$ (x in km). Find the fare for 5 km.
7. A candle is $h = 20 - 2t$ cm. After how long is it gone?
8. Find the equation through $(2, 7)$ and $(4, 13)$.
9. Does $(1, 5)$ lie on $y = 2x + 3$?
10. Find the y -intercept of $2x + y = 6$.



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10.F2 Quadratic function



DEFINITIONS

Quadratic function	A function of the form $y = ax^2 + bx + c$ whose graph is a parabola.
Parabola	The U-shaped curve of a quadratic. Opens up if $a > 0$, down if $a < 0$.
Turning point	The lowest (minimum) or highest (maximum) point of the parabola.
Roots / x-intercepts	Where $y = 0$ — found by factorising or using the quadratic formula.

KEY FORMULAS

- **Standard form:** $y = ax^2 + bx + c$
- **Y-intercept:** set $x = 0$, so $y = c$
- **X-intercepts (roots):** set $y = 0$ and factorise or use the formula
- **Difference of squares:** $x^2 - k^2 = (x - k)(x + k)$
- **Turning point (vertex):** when $b = 0$, it sits at $(0, c)$
- **Domain:** $x \in \mathbb{R}$
- **Range:** $y \geq c$ (if $a > 0$ and $b = 0$)

WORKED EXAMPLES

Example 1: Solve $x^2 - 9 = 0$ by factorisation.

- Recognise the difference of squares: $x^2 - 9 = (x - 3)(x + 3)$.
 $9 = 3^2$, so $k = 3$.
- Set each factor to zero: $x - 3 = 0 \rightarrow x = 3$, and $x + 3 = 0 \rightarrow x = -3$.
Two roots, symmetric about 0.

Example 2: For $y = x^2 - 4$, find the y-intercept and turning point.

- **Y-intercept:** set $x = 0 \rightarrow y = -4$.
Constant term gives y-intercept.
- **Turning point:** $b = 0$, so axis of symmetry is $x = 0$. $y(0) = -4$. TP = **(0, -4)**.
Same as the y-intercept here because $b = 0$.



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Example 3: For $y = x^2 - 25$, find the x-intercepts.

► Set $y = 0$: $x^2 - 25 = 0$.

Find where the parabola meets the x-axis.

► Factor: $(x - 5)(x + 5) = 0$.

Difference of squares with $k = 5$.

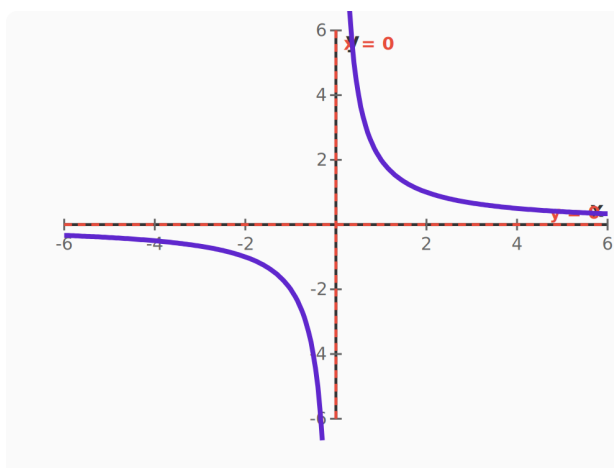
► $x = 5$ or $x = -5$.

Two symmetric roots.

EXERCISES (EASY → DIFFICULT)

- Find the y-intercept of $y = x^2 - 4$.
- Find the x-intercepts of $y = x^2 - 9$.
- Find the turning point of $y = x^2$.
- Does $y = x^2$ open up or down?
- Find the axis of symmetry of $y = x^2 - 4x$.
- A ball: $h = -x^2 + 4x$. Find where it lands ($h = 0$).
- Find the turning point of $y = x^2 - 4x + 3$.
- Does $y = -x^2 + 2$ have a max or min? Give it.
- Find the x-intercepts of $y = x^2 - 5x + 6$.
- $y = x^2$ is shifted up 3. Write the new equation.

10.F3 Hyperbolic function


DEFINITIONS
Hyperbola

The graph of $y = a/x$ — two curves in opposite quadrants.

Asymptote

A line the graph approaches but never touches. For $y = a/x$: vertical $x = 0$, horizontal $y = 0$.

KEY FORMULAS

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- **Standard form:** $y = a/x$
- **Vertical asymptote:** $x = 0$ (denominator can't be zero)
- **Horizontal asymptote:** $y = 0$ (the fraction $\rightarrow 0$ as $x \rightarrow \pm\infty$)
- **Quadrants:** $a > 0 \rightarrow Q1$ and $Q3$; $a < 0 \rightarrow Q2$ and $Q4$
- **Domain:** $x \neq 0$
- **Range:** $y \neq 0$

WORKED EXAMPLES

Example 1: For $y = 2/x$, what are the asymptotes?

- **Vertical:** $x = 0$ (we cannot divide by zero).

Denominator restriction.

- **Horizontal:** $y = 0$ (as $x \rightarrow \pm\infty$, $2/x \rightarrow 0$).

Limit at infinity.

Example 2: For $y = 6/x$, in which quadrants is the graph?

- Test $x = 1$: $y = 6 \rightarrow$ in **Q1** (x positive, y positive).

Plug in a positive x .

- Test $x = -1$: $y = -6 \rightarrow$ in **Q3** (x negative, y negative).

Plug in a negative x . Confirmed: $a > 0 \rightarrow Q1 + Q3$.

Example 3: For $y = -3/x$, in which quadrants is the graph?

- Test $x = 1$: $y = -3 \rightarrow$ in **Q4** (x positive, y negative).

Negative a flips quadrants.

- Test $x = -1$: $y = 3 \rightarrow$ in **Q2** (x negative, y positive).

$a < 0 \rightarrow Q2$ and $Q4$.

EXERCISES (EASY $\rightarrow$ DIFFICULT)

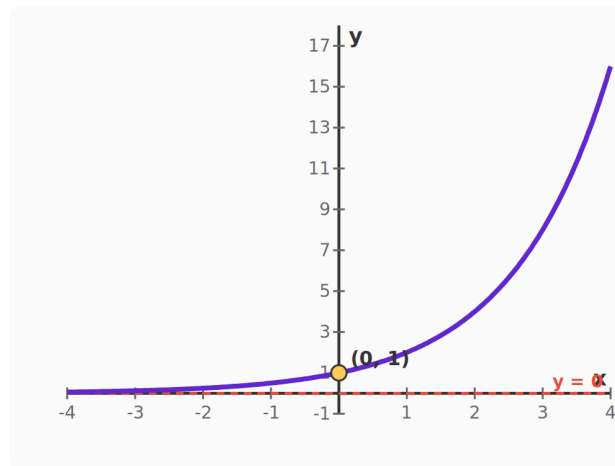
1. State the asymptotes of $y = 1/x$.
2. Find y when $x = 2$ for $y = 6/x$.
3. Find x when $y = 2$ for $y = 8/x$.
4. State the asymptotes of $y = 1/x + 2$.
5. As x grows large, $y = 1/x$ approaches...
6. A job takes $y = 240/x$ hours for x workers. Time for 8 workers?
7. Using $y = 240/x$, how many workers finish in 10 hours?
8. Find where $y = 4/x$ meets $y = x$.
9. Write $y = a/x$ through $(3, 4)$.
10. State the asymptotes of $y = 2/(x - 1)$.



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10.F4 Exponential function



DEFINITIONS

Exponential function	A function of the form $y = b^x$ where the variable is in the EXPONENT.
Base (b)	The constant being raised to the power x. Must be positive and $\neq 1$.
Horizontal asymptote	For $y = b^x$ this is $y = 0$ — the curve approaches but never touches the x-axis.

KEY FORMULAS

- **Standard form:** $y = b^x$, where $b > 0$ and $b \neq 1$
- **Y-intercept:** always $(0, 1)$ — because $b^0 = 1$
- **Horizontal asymptote:** $y = 0$
- **No x-intercept:** b^x never equals zero
- **Growth:** $b > 1 \rightarrow$ curve rises steeply right
- **Domain:** $x \in \mathbb{R}$
- **Range:** $y > 0$

WORKED EXAMPLES

Example 1: For $y = 2^x$, find y when $x = 0, 1, 2, 3$.

► $x=0 \rightarrow y = 2^0 = 1$. $x=1 \rightarrow y = 2$. $x=2 \rightarrow y = 4$. $x=3 \rightarrow y = 8$.

Each step doubles the previous value.

► Notice: y values **multiply by 2** each time x increases by 1.

Exponential growth pattern.

Example 2: For $y = 3^x$, find y when $x = -1$ and $x = 0$.

► $x = 0 \rightarrow y = 3^0 = 1$.

All exponentials hit $(0, 1)$.

► $x = -1 \rightarrow y = 3^{-1} = 1/3$.

Negative exponents give reciprocals.



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**Example 3: Why does $y = 4^x$ never cross the x-axis?**

- 4^x is always positive, no matter how negative x gets.

Even 4^{-100} is a tiny positive number.

- So $y > 0$ always. The line $y = 0$ is an **asymptote** the graph approaches but never touches.

Range is $y > 0$ (strictly positive).

EXERCISES (EASY → DIFFICULT)

1. State the asymptote of $y = 2^x$.
2. Find y when $x = 3$ for $y = 2^x$.
3. Find the y-intercept of $y = 2^x$.
4. Is $y = (\frac{1}{2})^x$ increasing or decreasing?
5. State the asymptote of $y = 3^x - 4$.
6. A colony $N = 2^t$ (hours). Find N after 5 hours.
7. A R1 000 investment $A = 1000(1.08)^t$. Value after 3 years?
8. A car $V = 200000(0.85)^t$. Value after 2 years?
9. Find the y-intercept of $y = 3 \cdot 2^x$.
10. Medicine $M = 50(0.5)^t$ mg. How much after 3 hours?

End of Chapter 11 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Find the gradient and y-intercept of $y = 2x + 3$.
2. Find the x-intercepts of $y = x^2 - 9$.
3. Find x when $y = 2$ for $y = 8/x$.
4. Is $y = (\frac{1}{2})^x$ increasing or decreasing?
5. State the gradient of $y = -\frac{1}{2}x + 5$.
6. A ball: $h = -x^2 + 4x$. Find where it lands ($h = 0$).
7. Using $y = 240/x$, how many workers finish in 10 hours?
8. A car $V = 200000(0.85)^t$. Value after 2 years?
9. Does $(1, 5)$ lie on $y = 2x + 3$?
10. $y = x^2$ is shifted up 3. Write the new equation.



CHAPTER 12

Exponents

10.E1 Revision of exponent laws

10.E2 Rational exponents

10.E3 Exponential equations

10.E1 Revision of exponent laws

$$(a^m)^n = a^{m \times n}$$

Multiply the exponents.

DEFINITIONS

Base	The number being repeatedly multiplied (e.g. in 2^3 , the base is 2).
Exponent (index)	The number telling you how many times to multiply (e.g. in 2^3 , the exponent is 3).
Product law	$a^m \times a^n = a^{m+n}$ — same base, ADD the exponents.
Quotient law	$a^m \div a^n = a^{m-n}$ — same base, SUBTRACT the exponents.
Power law	$(a^m)^n = a^{mn}$ — power of a power, MULTIPLY the exponents.

KEY FORMULAS

- **Product rule (same base):** $a^m \times a^n = a^{m+n}$
- **Quotient rule (same base):** $a^m \div a^n = a^{m-n}$
- **Power of a power:** $(a^m)^n = a^{m \times n}$
- **Power of a product:** $(a \cdot b)^n = a^n \cdot b^n$
- **Zero exponent:** $a^0 = 1$ (for $a \neq 0$)
- **Negative exponent:** $a^{-n} = 1 / a^n$



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WORKED EXAMPLES

Example 1: Simplify $a^5 \times a^3$.

► **Same base:** use product rule. Add exponents: $5 + 3 = 8$.

Apply $a^m \times a^n = a^{m+n}$.

► Result: **a^8** .

Just write the new exponent.

Example 2: Simplify $b^{10} \div b^4$.

► **Same base:** use quotient rule. Subtract exponents: $10 - 4 = 6$.

Apply $a^m/a^n = a^{m-n}$.

► Result: **b^6** .

Done.

Example 3: Simplify $(x^3y^2)^4$.

► Apply the power to EACH factor inside.

$(a \cdot b)^n = a^n \cdot b^n$.

► x^3 becomes $x^{3 \times 4} = x^{12}$. y^2 becomes $y^{2 \times 4} = y^8$.

Power-of-power on each factor.

► Result: **$x^{12}y^8$** .

Combine.

EXERCISES (EASY → DIFFICULT)

1. Simplify $x^3 \cdot x^4$.
2. Simplify $x^7 \div x^2$.
3. Simplify $(x^2)^3$.
4. Simplify x^0 ($x \neq 0$).
5. Simplify $(2x)^3$.
6. Simplify $2^3 \cdot 2^4$.
7. Simplify $x^5 \cdot x^{-2}$.
8. Simplify $(3x^2y)^2$.
9. Write x^{-3} with a positive exponent.
10. Simplify $(a^2b^3)(a^3b)$.



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10.E2 Rational exponents

$$a^{m/n} = \sqrt[n]{a^m}$$

apply the exponent laws

DEFINITIONS

Rational exponent	An exponent that is a fraction, e.g. $a^{1/n}$ means the n -th root of a .
$a^{1/n}$	The n -th root of a : $a^{1/2} = \sqrt{a}$, $a^{1/3} = \sqrt[3]{a}$.
$a^{(m/n)}$	$(\sqrt[n]{a})^m$ — take the n -th root, then raise to the m -th power.

KEY FORMULAS

- **Fractional exponent (1/n):** $a^{1/n} = \sqrt[n]{a}$ (the n -th root)
- **Rational exponent (m/n):** $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- **Two routes give the same answer:** root first then power, or power first then root
- **Negative rational exponent:** $a^{-m/n} = 1 / a^{m/n}$
- **Common roots:** $a^{1/2} = \sqrt{a}$, $a^{1/3} = \sqrt[3]{a}$

WORKED EXAMPLES

Example 1: Simplify $25^{1/2}$.

- **1/2 means square root.**

$$a^{1/2} = \sqrt{a}.$$

$$\sqrt{25} = 5 \text{ (since } 5^2 = 25\text{)}.$$

Find the number whose square is 25.

Example 2: Simplify $27^{1/3}$.

- **1/3 means cube root.**

$$a^{1/3} = \sqrt[3]{a}.$$

$$\sqrt[3]{27} = 3 \text{ (since } 3^3 = 27\text{)}.$$

Find the number whose cube is 27.

Example 3: Simplify $8^{2/3}$.

- **Take the cube root first:** $8^{1/3} = \sqrt[3]{8} = 2$.

Roots before powers keeps numbers small.

- **Now raise to power 2:** $2^2 = 4$.

Apply the m on top.

- **Result: 4.**

Same as $\sqrt[3]{(8^2)} = \sqrt[3]{64} = 4$.

EXERCISES (EASY → DIFFICULT)

1. Write $\sqrt{x}$ as a power.
2. Evaluate $9^{(1/2)}$.
3. Evaluate $8^{(1/3)}$.



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4. Evaluate $16^{(3/4)}$.
5. Evaluate $25^{(1/2)}$.
6. Evaluate $27^{(1/3)}$.
7. Write $\sqrt[3]{(x^2)}$ as a power.
8. Evaluate $4^{(3/2)}$.
9. Evaluate $32^{(2/5)}$.
10. Simplify $x^{(1/2)} \cdot x^{(1/2)}$.

10.E3 Exponential equations

solve for the exponent

apply the exponent laws

DEFINITIONS

Exponential equation

An equation where the unknown is in the EXPONENT, e.g. $2^x = 32$.

Strategy

Rewrite both sides as a power of the SAME BASE, then equate the exponents.

KEY FORMULAS

- **Equal-base rule:** if $a^x = a^k$ (with $a > 0$, $a \neq 1$), then $x = k$
- **Strategy:** rewrite both sides with the SAME base, then equate exponents
- **Useful powers:** $2^5 = 32$, $3^3 = 27$, $2^{10} = 1024$, $4^3 = 64$
- **Compound exponent:** if $2^{x-1} = 8$, solve $x - 1$ first, then x

WORKED EXAMPLES

Example 1: Solve $2^x = 16$.

- Rewrite 16 as a power of 2: $16 = 2^4$.

Match the bases.

- Now $2^x = 2^4$. Equate exponents: $x = 4$.

Equal bases means equal exponents.

Example 2: Solve $2^{x-1} = 8$.

- $8 = 2^3$. So $2^{x-1} = 2^3$.

Same-base form.

- Equate exponents: $x - 1 = 3$, so $x = 4$.

Solve the resulting equation.

Example 3: Solve $3^{2x} = 27$.

- $27 = 3^3$.

Recognise the cube.

- $3^{2x} = 3^3 \Rightarrow 2x = 3 \Rightarrow x = 1.5$.

Solve for x .



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**EXERCISES (EASY → DIFFICULT)**

1. Solve $2^x = 8$.
2. Solve $3^x = 81$.
3. Solve $2^x = 32$.
4. Solve $5^x = 1$.
5. Solve $2^x = 16$.
6. Solve $10^x = 1000$.
7. Solve $2^{(x+1)} = 16$.
8. Solve $3^{(2x)} = 81$.
9. A colony $N = 2^t$ reaches 64 after how many hours?
10. Solve $4^x = 64$.

End of Chapter 12 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Simplify $x^3 \cdot x^4$.
2. Evaluate $9^{(1/2)}$.
3. Solve $2^x = 32$.
4. Simplify x^0 ($x \neq 0$).
5. Evaluate $25^{(1/2)}$.
6. Solve $10^x = 1000$.
7. Simplify $x^5 \cdot x^{-2}$.
8. Evaluate $4^{(3/2)}$.
9. A colony $N = 2^t$ reaches 64 after how many hours?
10. Simplify $(a^2b^3)(a^3b)$.

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CHAPTER 13

Number Patterns

10.P1 Arithmetic sequences

10.P1 Arithmetic sequences



DEFINITIONS

Sequence	A list of numbers in a definite order, e.g. 3, 7, 11, 15, ...
Arithmetic sequence	A sequence where you ADD the same constant (the COMMON DIFFERENCE) to get the next term.
Common difference (d)	$d = T_2 - T_1$ (any consecutive pair gives the same answer).
General term (T_n)	A formula for the n-th term: $T_n = a + (n - 1) \cdot d$, where a is the first term.

KEY FORMULAS

- **Arithmetic sequence:** each term adds a fixed common difference d
- **Common difference:** $d = T_2 - T_1 = T_3 - T_2 = \dots$
- **n-th term formula:** $T_n = a + (n - 1)d$
- **where** $a = T_1$ (first term), n = position number, d = common difference
- **Spotting:** the differences between consecutive terms are all the same

WORKED EXAMPLES

Example 1: For 3, 7, 11, 15, ... find a, d, and T_{10} .

► $a = 3$ (first term). $d = 7 - 3 = 4$.

Read off a; subtract consecutive terms for d.

► $T_{10} = a + (10 - 1)d = 3 + 9 \times 4 = 3 + 36 = 39$.

Plug $n=10$ into the formula.



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**Example 2: For 5, 9, 13, 17, ... find T_8 .**

► $a = 5, d = 4.$

First term and difference.

► $T_8 = 5 + 7 \times 4 = 5 + 28 = 33.$

Plug $n=8$ into the formula.

Example 3: For 10, 7, 4, 1, ... find T_6 (decreasing sequence).

► $a = 10, d = 7 - 10 = -3.$

Negative d means the sequence decreases.

► $T_6 = 10 + 5 \times (-3) = 10 - 15 = -5.$

Multiply $(n-1)$ by d , even when negative.

EXERCISES (EASY → DIFFICULT)

1. Find the next term of 3, 7, 11, 15, ...
2. Find the common difference of 5, 9, 13, ...
3. For 2, 5, 8, ... find T_{10} .
4. Find T_n for 1, 6, 11, 16, ...
5. Find the 20th term of 4, 7, 10, ...
6. A theatre has 20 seats in row 1 and 4 more each row. Seats in row 10?
7. Lerato's salary starts at R8 000, rising R500/year. Salary in year 6?
8. Which term of 4, 7, 10, ... equals 100?
9. Find T_n for 7, 4, 1, -2, ...
10. The 3rd term is 11 and the 7th is 27. Find a and d .

End of Chapter 13 — Mixed Exercise

Ten questions across the whole chapter, from easy to difficult.

1. Find the next term of 3, 7, 11, 15, ...
2. Find the common difference of 5, 9, 13, ...
3. For 2, 5, 8, ... find T_{10} .
4. Find T_n for 1, 6, 11, 16, ...
5. Find the 20th term of 4, 7, 10, ...
6. A theatre has 20 seats in row 1 and 4 more each row. Seats in row 10?
7. Lerato's salary starts at R8 000, rising R500/year. Salary in year 6?
8. Which term of 4, 7, 10, ... equals 100?
9. Find T_n for 7, 4, 1, -2, ...
10. The 3rd term is 11 and the 7th is 27. Find a and d .

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ANSWERS

Worked solutions to all exercises

Each answer gives the final result first, followed by the key working steps. For fully animated, step-by-step solutions, open the digital edition with the QR code on any page.

CHAPTER 1 — Algebraic expressions

1.2 The real number system

1. Yes — natural numbers are 1, 2, 3, ...
2. Yes (whole numbers include 0).
3. Yes.
4. Rational (a fraction of integers).
5. Irrational.
6. $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$ (not $\mathbb{N}$).
7. e.g. π or $\sqrt{2}$.
8. Yes $\rightarrow 3/4$.
9. Yes ($n = n/1$).
10. $4 \in \mathbb{N}$; $-2 \in \mathbb{Z}$; $\frac{2}{3} \in \mathbb{Q}$; $\sqrt{9} = 3 \in \mathbb{N}$.

1.3 Rational and irrational numbers

1. Rational.
2. $\sqrt{16} = 4 \rightarrow$ rational.
3. Irrational.
4. $1/3$.
5. Irrational.
6. 3 and 4 (since $3^2 = 9$, $4^2 = 16$).
7. No — 7 is not a perfect square, so $\sqrt{7}$ is irrational.
8. $45/99 = 5/11$.
9. Rational (a fraction) — note it only approximates π .
10. Let $x = 0.999\dots$; $10x = 9.999\dots$; $10x - x = 9 \rightarrow 9x = 9 \rightarrow x = 1$.

1.4 Rounding off

1. 3.47.
2. 12.8.
3. 0.05.
4. 150.
5. 3.142.
6. R47.38.
7. 2.85 m.
8. 10 000.
9. R23.46.
10. $0.6 \times 349 = 209.4 \rightarrow$ R209.





1.5 Estimating surds

1. 4 and 5.
2. 7 and 8.
3. ≈ 5 .
4. 2 and 3.
5. ≈ 8 .
6. $\sqrt{40} \approx 6.32 > 6$.
7. 11 and 12.
8. $\sqrt{45} \approx 7$ m.
9. $2 (= \sqrt{4}) < \sqrt{5} < \sqrt{8}$.
10. 9 and 10 ($\sqrt{90} \approx 9.49$).

1.6 Products: distributive

1. $3x + 12$.
2. $10a - 15$.
3. $-2x + 14$.
4. $x^2 + 6x$.
5. $8x^2 - 4x + 12$.
6. $6a^2 + 10a$.
7. $-3x^2 + 6xy$.
8. $2x + 6 + 3x - 3 = 5x + 3$.
9. $5x - 10 - 2x - 8 = 3x - 18$.
10. $4(x + 3) = 4x + 12$.

1.6 Products: binomial $\times$ binomial (FOIL)

1. $x^2 + 7x + 10$.
2. $x^2 - x - 12$.
3. $x^2 - 8x + 12$.
4. $2x^2 + 7x + 3$.
5. $3x^2 + 10x - 8$.
6. $2x^2 + 9x - 5$.
7. $6x^2 - 11x + 3$.
8. $x^2 + 5x + 4$.
9. $x^2 - 4$.
10. $x^2 + 5x + 6 - x^2 = 5x + 6$.

1.6 Products: difference of squares

1. $x^2 - 9$.
2. $x^2 - 25$.
3. $4x^2 - 1$.
4. $9x^2 - 16$.
5. $a^2 - 49$.
6. $(100+1)(100-1) = 10000 - 1 = 9999$.
7. $25 - x^2$.
8. $(50+3)(50-3) = 2500 - 9 = 2491$.
9. $x^2y^2 - 4$.
10. $4a^2 - 9b^2$.

1.6 Products: perfect square trinomial

1. $x^2 + 6x + 9$.
2. $x^2 - 8x + 16$.
3. $4x^2 + 4x + 1$.
4. $9x^2 - 12x + 4$.
5. $x^2 + 10x + 25$.
6. $400 + 40 + 1 = 441$.





7. $a^2 - 2ab + b^2$.
8. $10000 - 200 + 1 = 9801$.
9. $4x^2 + 12xy + 9y^2$.
10. $x^2 + 12x + 36$.

1.7 Factorisation: common factor

1. $3(2x + 3)$.
2. $4(a - 2)$.
3. $x(x + 5)$.
4. $4y(3y - 2)$.
5. $5ab(3a + 2b)$.
6. $3(x^2 + 2x + 3)$.
7. $-2x(x + 2)$.
8. $7x(x^2 - 2x + 3)$.
9. $(x + 1)(a + b)$.
10. $2\pi(r + R)$.

1.7 Factorisation: difference of squares

1. $(x + 4)(x - 4)$.
2. $(x + 7)(x - 7)$.
3. $(2x + 3)(2x - 3)$.
4. $(5 + x)(5 - x)$.
5. $(3x + 4)(3x - 4)$.
6. $(a + b)(a - b)$.
7. $2(x^2 - 9) = 2(x + 3)(x - 3)$.
8. $(x^2 + 1)(x + 1)(x - 1)$.
9. $(7a + 9b)(7a - 9b)$.
10. $(x + \frac{1}{2})(x - \frac{1}{2})$.

1.7 Factorisation: trinomials $x^2 + bx + c$

1. $(x + 2)(x + 3)$.
2. $(x + 3)(x + 4)$.
3. $(x + 3)(x + 5)$.
4. $(x + 2)(x + 4)$.
5. $(x + 4)(x + 5)$.
6. $(x + 1)^2$.
7. $(x + 3)(x + 7)$.
8. $(x + 3)(x + 8)$.
9. $(x + 5)(x + 8)$.
10. $(x + 2)$ and $(x + 5)$.

1.7 Factorisation: trinomials with negative middle

1. $(x - 2)(x - 3)$.
2. $(x - 3)(x - 4)$.
3. $(x - 3)(x + 2)$.
4. $(x + 4)(x - 2)$.
5. $(x - 5)(x + 2)$.
6. $(x - 4)(x - 5)$.
7. $(x + 4)(x - 3)$.
8. $(x - 7)(x + 3)$.
9. $(x - 5)(x + 3)$.
10. $(x - 2)(x - 4)$.





1.7 Factorisation: grouping (4 terms)

1. $x^2(x + 1) + 2(x + 1) = (x + 1)(x^2 + 2)$.
2. $a(x + y) + b(x + y) = (x + y)(a + b)$.
3. $(x + y)(2 + a)$.
4. $x^2(x - 3) + 2(x - 3) = (x - 3)(x^2 + 2)$.
5. $x(3 + y) + 1(3 + y) = (3 + y)(x + 1)$.
6. $a(b - 2) + 3(b - 2) = (b - 2)(a + 3)$.
7. $x(x + y) + 5(x + y) = (x + y)(x + 5)$.
8. $2(3 + x) + y(3 + x) = (3 + x)(2 + y)$.
9. $2a(x - 2) + 3(x - 2) = (x - 2)(2a + 3)$.
10. $p(q - 1) - 1(q - 1) = (q - 1)(p - 1)$.

1.8 Simplification of fractions

1. $x/2$.
2. $2x$.
3. $(x + 3)$.
4. $(x + 3)$.
5. 4 .
6. $(x + 2)/(x - 2)$.
7. $5/x$.
8. $5/6$.
9. $x/(3y)$.
10. $(x + 2)/(x + 3)$.

End of Chapter 1 — Mixed Exercise

1. Yes — natural numbers are 1, 2, 3, ...
2. $\sqrt{16} = 4 \rightarrow$ rational.
3. 0.05.
4. 2 and 3.
5. $8x^2 - 4x + 12$.
6. $2x^2 + 9x - 5$.
7. $25 - x^2$.
8. $10000 - 200 + 1 = 9801$.
9. $(x + 1)(a + b)$.
10. $(x + \frac{1}{2})(x - \frac{1}{2})$.

CHAPTER 4 — Equations and inequalities

4.2 Solving linear equations (one-step)

1. $x = 7$.
2. $x = 12$.
3. $x = 5$.
4. $x = 18$.
5. $x = -5$.
6. $x = 7$.
7. $x - 4 = 11 \rightarrow x = 15$.
8. $3x = 27 \rightarrow x = 9$.
9. $x = -7$.
10. $x - 8 = 12 \rightarrow R20$.





4.2 Solving linear equations (two-step)

1. $2x = 8 \rightarrow x = 4.$
2. $3x = 21 \rightarrow x = 7.$
3. $x = 3.$
4. $x = 4.$
5. $x/2 = 4 \rightarrow x = 8.$
6. $-2x = -6 \rightarrow x = 3.$
7. $3x + 5 = 20 \rightarrow x = 5 \text{ km}.$
8. $x = 4.$
9. $2x + 50 = 80 \rightarrow x = 15.$
10. $2x - 1 = 15 \rightarrow x = 8.$

4.2 Linear with brackets

1. $2x + 6 = 14 \rightarrow x = 4.$
2. $x - 2 = 4 \rightarrow x = 6.$
3. $8x - 4 = 20 \rightarrow x = 3.$
4. $5x + 5 = 3x + 15 \rightarrow 2x = 10 \rightarrow x = 5.$
5. $2x - 6 + 4 = 10 \rightarrow 2x = 12 \rightarrow x = 6.$
6. $6x + 3 - 2x + 2 = 17 \rightarrow 4x = 12 \rightarrow x = 3.$
7. $-x + 4 = 2 \rightarrow x = 2.$
8. $6x - 2 = 4x + 8 \rightarrow 2x = 10 \rightarrow x = 5.$
9. $4x + 6 = 26 \rightarrow x = 5.$
10. $4x + 8 - 3x + 3 = 15 \rightarrow x = 4.$

4.2 Variable on both sides

1. $3x = 9 \rightarrow x = 3.$
2. $2x = 10 \rightarrow x = 5.$
3. $3x = 12 \rightarrow x = 4.$
4. $3x = 12 \rightarrow x = 4.$
5. $-3x = -9 \rightarrow x = 3.$
6. $8 = 4x \rightarrow x = 2.$
7. $3x + 3 = x + 9 \rightarrow 2x = 6 \rightarrow x = 3.$
8. $x = 2x - 6 \rightarrow x = 6.$
9. $3x = 9 \rightarrow x = 3.$
10. $9 = 3x \rightarrow x = 3.$

4.3 Solve quadratic by factorising

1. $(x - 2)(x - 3) = 0 \rightarrow x = 2 \text{ or } 3.$
2. $(x + 5)(x - 2) = 0 \rightarrow x = -5 \text{ or } 2.$
3. $x = 3 \text{ or } 4.$
4. $x = 2 \text{ or } -3.$
5. $x = 4 \text{ or } -3.$
6. $x(x + 2) = 0 \rightarrow x = 0 \text{ or } -2.$
7. $x = 0 \text{ or } 4.$
8. $n(n+1)=30 \rightarrow n^2+n-30=0 \rightarrow 5 \text{ and } 6.$
9. $(x + 3)^2 = 0 \rightarrow x = -3.$
10. $w(w+1)=20 \rightarrow w=4.$

4.3 Quadratic with difference of squares

1. $(x + 3)(x - 3) = 0 \rightarrow x = \pm 3.$
2. $x = \pm 5.$
3. $(2x + 1)(2x - 1) = 0 \rightarrow x = \pm 1/2.$
4. $x = \pm 4.$
5. $x = \pm 2/3.$
6. $x = \pm 7.$





7. $x^2 = 4 \rightarrow x = \pm 2$.
8. $x = \pm 10$.
9. $x^2 = 36 \rightarrow x = 6 \text{ cm}$.
10. $x = \pm 3/4$.

4.4 Solving simultaneous equations

1. $x + 1 = 2x - 1 \rightarrow x = 2, y = 3$.
2. Add: $2x = 8 \rightarrow x = 4, y = 3$.
3. $2x + x - 1 = 8 \rightarrow x = 3, y = 2$.
4. Subtract: $x = 6, y = 4$.
5. $6 + 2y = 12 \rightarrow y = 3$.
6. $x + y = 12, x - y = 4 \rightarrow 8 \text{ and } 4$.
7. $y + 2 + 2y = 11 \rightarrow y = 3, x = 5$.
8. $60 + c = 80 \rightarrow c = \text{R}20$.
9. Add: $5x = 10 \rightarrow x = 2, y = 3$.
10. From first $y = 9 - x \rightarrow 2x + 45 - 5x = 30 \rightarrow x = 5, y = 4$.

4.5 Word problems

1. $x + 7 = 19 \rightarrow x = 12$.
2. $2x - 3 = 11 \rightarrow x = 7$.
3. $3n + 3 = 36 \rightarrow 11, 12, 13$.
4. $s + 3s = 48 \rightarrow s = 12$.
5. $2(w + w + 5) = 38 \rightarrow w = 7$.
6. $3x + 12 = 27 \rightarrow x = \text{R}5$.
7. $x + 4x = 40 \rightarrow 8 \text{ and } 32$.
8. $4x + 6 = 34 \rightarrow x = 7 \text{ km}$.
9. $3b = 30 \rightarrow b = 10$.
10. $20w + 50 = 210 \rightarrow w = 8$.

4.6 Literal equations

1. $x = y - a$.
2. $x = y/2$.
3. $x = (y - c)/m$.
4. $r = C/(2\pi)$.
5. $h = 2A/b$.
6. $x = (c - b)/a$.
7. $t = (v - u)/a$.
8. $b = (P - 2a)/2$.
9. $r = \sqrt{A/\pi}$.
10. $x = 4y + 3$.

4.7 Solve linear inequality

1. $x > 4$.
2. $x < 5$.
3. $x \geq 5$.
4. $3x \leq 9 \rightarrow x \leq 3$.
5. $x > 3$.
6. $x \geq 8$.
7. Open circle at 2, shade to the right.
8. $x < 3$.
9. $x \geq 40$.
10. $x \leq 3$.





4.7 Inequality with negative coefficient

1. $x < -3$ (flip the sign).
2. $x > -4$.
3. $x \leq -3$.
4. $-x > -3 \rightarrow x < 3$.
5. $x \geq -8$.
6. $-2x \geq 6 \rightarrow x \leq -3$.
7. Dividing or multiplying by a negative reverses the inequality.
8. $-4x < 8 \rightarrow x > -2$.
9. $-5x \leq -15 \rightarrow x \geq 3$.
10. $-3x > -6 \rightarrow x < 2$.

End of Chapter 4 — Mixed Exercise

1. $x = 7$.
2. $3x = 21 \rightarrow x = 7$.
3. $8x - 4 = 20 \rightarrow x = 3$.
4. $3x = 12 \rightarrow x = 4$.
5. $x = 4$ or -3 .
6. $x = \pm 7$.
7. $y + 2 + 2y = 11 \rightarrow y = 3, x = 5$.
8. $4x + 6 = 34 \rightarrow x = 7$ km.
9. $r = \sqrt{A/\pi}$.
10. $x \leq 3$.

CHAPTER 7 — Euclidean geometry

7.2 Triangle angle sum

1. 70° .
2. 55° .
3. 60° .
4. $2x = 100 \rightarrow x = 50^\circ$.
5. $9x = 180 \rightarrow x = 20^\circ$.
6. 70° .
7. $3x + 60 = 180 \rightarrow x = 40^\circ$.
8. $3/6 \times 180 = 90^\circ$.
9. $2\angle C + \angle C = 90 \rightarrow \angle C = 30^\circ$.
10. 40° .

7.2 Exterior angle of triangle

1. 70° .
2. remote (opposite) interior angles.
3. 100° .
4. 50° .
5. 120° .
6. $2x = 90 \rightarrow x = 45^\circ$.
7. $180 - 130 = 50^\circ$.
8. 45° .
9. Interior angles sum to 180° , and the exterior angle is 180° minus its adjacent interior angle.
10. 47.5° .





7.2 Congruency tests (SSS, SAS, AAS, RHS)

1. SSS.
2. SAS.
3. AAS.
4. RHS.
5. Yes (SSS).
6. SAS.
7. Yes (RHS).
8. AAS (a form of ASA/AAS).
9. No — only similarity.
10. SAS fixes the triangle uniquely; SSA can give two different triangles.

7.2 Similarity tests (AA, SAS, SSS-proportional)

1. AA.
2. SSS (proportional).
3. SAS (proportional).
4. angles; proportion.
5. Yes (ratio 1:2, SSS).
6. $\frac{2}{3} = \frac{h}{18} \rightarrow h = 12$ m.
7. Yes (AA).
8. $5 \times 3^2 = 45$.
9. $\frac{4}{6} = \frac{12}{L} \rightarrow L = 18$.
10. $4 \times 50000 = 200000$ cm = 2 km.

7.3 Quadrilaterals

1. 360° .
2. 80° .
3. equal.
4. equal (and bisect each other).
5. equal.
6. $10x = 360 \rightarrow x = 36^\circ$.
7. 180° .
8. right angles (and a rectangle with equal sides).
9. $70^\circ, 110^\circ, 110^\circ$.
10. 90° (right angles).

7.4 Mid-point theorem

1. parallel to and half.
2. 6.
3. 14.
4. 5.
5. The third (un-joined) side.
6. 8.
7. 18.
8. 10 cm.
9. The small triangle is similar to the whole with ratio 1:2.
10. 13.

End of Chapter 7 — Mixed Exercise

1. 70° .
2. remote (opposite) interior angles.
3. AAS.
4. angles; proportion.
5. equal.
6. 8.





7. $3x + 60 = 180 \rightarrow x = 40^\circ$.
8. 45° .
9. No — only similarity.
10. $4 \times 50000 = 200000 \text{ cm} = 2 \text{ km}$.

CHAPTER 8 — Analytical geometry

8.2 Distance between two points

1. $\sqrt{9 + 16} = 5$.
2. 5.
3. 13.
4. 5.
5. 6.
6. 5 km.
7. 10 km.
8. 5.
9. 10 km.
10. Gradient $4/3$ throughout $\rightarrow$ collinear.

8.3 Gradient of a line

1. 3.
2. 3.
3. -2 .
4. 2.
5. 0.
6. $\frac{1}{2}$.
7. 2.
8. equal.
9. $1/5$.
10. -1 .

8.4 Mid-point of a line

1. (4, 6).
2. (2, 5).
3. (1, 1).
4. (4, 3).
5. (0, 0).
6. (3, 5).
7. (0, 0).
8. $(y + 10)/2 = 7 \rightarrow y = 4$.
9. (0, 3).
10. $Q = (5, 6)$.

End of Chapter 8 — Mixed Exercise

1. $\sqrt{9 + 16} = 5$.
2. 3.
3. (1, 1).
4. 5.
5. 0.
6. (3, 5).
7. 10 km.
8. equal.
9. (0, 3).
10. Gradient $4/3$ throughout $\rightarrow$ collinear.



CHAPTER 9 — Finance and growth

9.2 Simple interest

1. $I = 1000 \cdot 0.08 \cdot 1 = R80.00$.
2. $I = 2000 \cdot 0.10 \cdot 3 = R600.00$.
3. $A = 5000(1 + 0.06 \cdot 2) = R5,600.00$.
4. R160.00.
5. $A = P(1 + i \cdot n)$.
6. $A = 12000(1 + 0.075 \cdot 5) = R16,500.00$.
7. $240 = 1000 \cdot 0.08 \cdot n \rightarrow n = 3$ years.
8. R960.00.
9. $300 = 2000 \cdot i \cdot 2 \rightarrow i = 7.5\%$.
10. $A = 6000(1 + 0.27) = R7,620.00$.

9.3 Compound interest

1. $A = 1000(1.08)^2 = R1,166.40$.
2. $A = R2,662.00$.
3. $A = P(1 + i)^n$.
4. $A = R6,312.38$.
5. $A = R3,434.70$; interest = R434.70.
6. $A = R12,308.99$.
7. R12,597.12.
8. Simple R1,200.00 vs compound R1,210.00 — compound earns more.
9. R5,017.60.
10. $5832 = P(1.08)^2 \rightarrow P = R5\ 000$.

9.4 Calculations with simple/compound interest

1. R2,720.00.
2. R2,823.16.
3. Compound (interest earns interest).
4. R22,510.96.
5. R1,910.16.
6. R45,219.67.
7. R77,500.00.
8. $\log 2 / \log 1.1 \approx 7.3 \rightarrow$ about 8 years.
9. R9,034.13.
10. Simple R6,050.00 vs compound R6,125.22.

9.5 Foreign exchange rates

1. $100 \times 0.055 = \$5.50$.
2. $50 \times 18 = R900$.
3. $\text{£}20 \times 23 = R460$.
4. $400 \div 20 = \text{€}20$.
5. $1800 \div 18 = \$100$.
6. $40 \times 19 = R760$.
7. $2700 \div 27 = \text{£}100$.
8. $200 \times 18.50 = R3\ 700$.
9. $\$90 = R1\ 620 \rightarrow \div 20 = \text{€}81$.
10. $330 \times 19 = R6\ 270$.



**End of Chapter 9 — Mixed Exercise**

1. $I = 1000 \cdot 0.08 \cdot 1 = R80.00$.
2. $A = R2,662.00$.
3. Compound (interest earns interest).
4. $400 \div 20 = €20$.
5. $A = P(1 + i \cdot n)$.
6. $A = R12,308.99$.
7. $R77,500.00$.
8. $200 \times 18.50 = R3\,700$.
9. $300 = 2000 \cdot i \cdot 2 \rightarrow i = 7.5\%$.
10. $5832 = P(1.08)^2 \rightarrow P = R5\,000$.

CHAPTER 10 — Statistics**10.2 Mean**

1. 6.
2. 5.
3. 30.
4. 6.
5. 15.
6. 75.
7. $R250$.
8. 8.
9. 48.
10. total 30 $\rightarrow 30 - 17 = 13$.

10.2 Median

1. 5.
2. 5.
3. 3.
4. 25.
5. Order: 2,4,5,7,9 $\rightarrow 5$.
6. 60.
7. 17.
8. It is not affected by extreme values (outliers).
9. Order: 5,6,7,8,100 $\rightarrow 7$.
10. $(300 + 350)/2 = R325$.

10.2 Mode

1. 3.
2. 6.
3. No mode.
4. Two modes: 5 and 7 (bimodal).
5. 7.
6. The mode.
7. 12.
8. Red.
9. 8.
10. If every value occurs the same number of times.





10.3 Grouping data (frequency)

1. how many times a value (or class) occurs.
2. 15.
3. [20–29].
4. 7.
5. 0–4: 2 values; 5–9: 3 values.
6. 22.
7. 8.
8. To summarise large data sets and see patterns.
9. The class with frequency 10.
10. $50 - 36 = 14$.

10.4 Range

1. $15 - 2 = 13$.
2. 30.
3. 0.
4. $250 - 90 = 160$.
5. spread (extent).
6. $25 - 9 = 16^\circ$.
7. $90 - 30 = 60$.
8. {1,5,9} (range 8).
9. $20 - 8 = 12$.
10. It uses only the two extreme values and ignores the rest.

10.4 Inter-quartile range

1. 15.
2. $Q1 = 5, Q3 = 13 \rightarrow IQR = 8$.
3. 50%.
4. 40.
5. 2.5.
6. $Q1 = 55, Q3 = 75 \rightarrow 20$.
7. It ignores extreme outliers, using only the middle half.
8. $Q1 = 9, Q3 = 19 \rightarrow 10$.
9. 30.
10. Sorted: 3,5,7,8,12,13,14,18,21; $Q1=6, Q3=16 \rightarrow IQR = 10$.

10.5 Five-number summary

1. min, Q1, median, Q3, max.
2. min 3, Q1 5, med 8, Q3 13.5, max 15.
3. five-number summary.
4. min 40, Q1 55, med 65, Q3 75, max 90.
5. 20.
6. R425.
7. 50.
8. IQR 15; range 38.
9. min 10, Q1 13, med 17, Q3 21, max 24.
10. The line inside the box.

End of Chapter 10 — Mixed Exercise

1. 6.
2. 5.
3. No mode.
4. 7.
5. spread (extent).
6. $Q1 = 55, Q3 = 75 \rightarrow 20$.



7. 50.
8. 8.
9. Order: 5,6,7,8,100 $\rightarrow$ 7.
10. If every value occurs the same number of times.

CHAPTER 12 — Euclidean geometry: proofs

12.1 Conjecture about parallelogram diagonals

1. They bisect each other.
2. 5.
3. 6.
4. No (only in a rectangle).
5. congruent.
6. $2x = x + 4 \rightarrow x = 4$.
7. $AM = 5$, $BM = 4$.
8. They share the diagonal and have equal alternate angles (SAS/ASA).
9. parallelogram.
10. $3x - 1 = x + 5 \rightarrow x = 3 \rightarrow BM = 8$.

12.1 Conjecture about rhombus diagonals

1. They bisect each other at right angles (90°).
2. Yes.
3. 90° .
4. $\sqrt{3^2 + 4^2} = 5$.
5. Yes.
6. $\sqrt{5^2 + 12^2} = 13$.
7. two 40° angles.
8. $\frac{1}{2} \cdot 6 \cdot 8 = 24$.
9. The four triangles formed are congruent, forcing right angles at the centre.
10. $\text{half} = \sqrt{13^2 - 5^2} = 12 \rightarrow \text{other diagonal } 24$.

End of Chapter 12 — Mixed Exercise

1. They bisect each other.
2. Yes.
3. 6.
4. $\sqrt{3^2 + 4^2} = 5$.
5. congruent.
6. $\sqrt{5^2 + 12^2} = 13$.
7. $AM = 5$, $BM = 4$.
8. $\frac{1}{2} \cdot 6 \cdot 8 = 24$.
9. parallelogram.
10. $\text{half} = \sqrt{13^2 - 5^2} = 12 \rightarrow \text{other diagonal } 24$.

CHAPTER 13 — Measurement

13.1 Area of triangle

1. $\frac{1}{2} \cdot 10 \cdot 6 = 30$.
2. 20.
3. 42.
4. $h = 2 \cdot 24 / 8 = 6$.
5. $A = \frac{1}{2} \times \text{base} \times \text{height}$.
6. 12 m^2 .
7. 300 m^2 .
8. $b = 2 \cdot 50 / 10 = 10 \text{ cm}$.





9. 54.

10. 7.5 m^2 .

13.1 Area of trapezium

1. $\frac{1}{2}(6 + 10) \cdot 4 = 32$.

2. 42.

3. $A = \frac{1}{2}(a + b) \cdot h$.

4. 50.

5. $40 = \frac{1}{2} \cdot 16 \cdot h \rightarrow h = 5$.

6. 800 m^2 .

7. 72.

8. 0.5 m^2 .

9. $96 = \frac{1}{2}(10 + b) \cdot 8 \rightarrow b = 14$.

10. 200 cm^2 .

13.1 Area of circle ($\pi = 3.14$)

1. $3.14 \cdot 100 = 314$.

2. 78.5.

3. $A = \pi r^2$.

4. 12.56.

5. $r = 10 \rightarrow 314$.

6. $3.14 \cdot 0.49 = 1.54 \text{ m}^2$.

7. 706.5 cm^2 .

8. $r^2 = 9 \rightarrow r = 3$.

9. 50.24 m^2 .

10. 153.86 m^2 .

13.2 Volume of cylinder ($\pi = 3.14$)

1. $3.14 \cdot 100 \cdot 5 = 1\,570$.

2. 125.6.

3. $V = \pi r^2 h$.

4. 314.

5. 197.82.

6. 6.28 m^3 .

7. 1538.6 cm^3 .

8. $1256 = 314 \cdot h \rightarrow h = 4$.

9. $0.785 \text{ m}^3 = 785 \square$.

10. 1256 cm^3 .

13.3 Volume of sphere

1. $\frac{4}{3} \cdot 3.14 \cdot 27 = 113.04$.

2. $V = \frac{4}{3} \cdot \pi r^3$.

3. 904.32.

4. 4.19.

5. 33.49.

6. 5572.45 cm^3 .

7. 0.52 m^3 .

8. 4.19 cm^3 .

9. $r^3 = 8 \rightarrow r = 2$.

10. 33493.33 cm^3 .





13.3 Volume of cone

1. $\frac{1}{3} \cdot 3.14 \cdot 9 \cdot 6 = 56.52$.
2. $V = \frac{1}{3} \cdot \pi r^2 h$.
3. 235.5.
4. 50.24.
5. 376.8.
6. 113.04 cm^3 .
7. 6.28 m^3 .
8. $100.48 = \frac{1}{3} \cdot 3.14 \cdot 16 \cdot h \rightarrow h = 6$.
9. 1230.88.
10. One third.

13.4 Effect of multiplying dimensions by k

1. 4 times (2^2).
2. 8 times (2^3).
3. 9 times.
4. 27 times.
5. $\frac{1}{8}$.
6. 8 times.
7. 100.
8. $1.5^3 = 3.375$.
9. $k^3 = 8 \rightarrow k = 2$.
10. $12 \times 16 = 192 \text{ cm}^2$.

End of Chapter 13 — Mixed Exercise

1. $\frac{1}{2} \cdot 10 \cdot 6 = 30$.
2. 42.
3. $A = \pi r^2$.
4. 314.
5. 33.49.
6. 113.04 cm^3 .
7. 100.
8. $b = 2 \cdot 50 / 10 = 10 \text{ cm}$.
9. $96 = \frac{1}{2}(10 + b) \cdot 8 \rightarrow b = 14$.
10. 153.86 m^2 .

CHAPTER 14 — Probability

14.1 Theoretical probability

1. $\frac{1}{2}$.
2. $\frac{1}{6}$.
3. $\frac{3}{6} = \frac{1}{2}$.
4. $\frac{3}{5}$.
5. $P(\text{event}) = \text{favourable outcomes} \div \text{total outcomes}$.
6. $\frac{2}{6} = \frac{1}{3}$.
7. $\frac{4}{52} = \frac{1}{13}$.
8. $\frac{3}{8}$.
9. $\frac{3}{10}$.
10. $\frac{12}{30} = \frac{2}{5}$.





14.2 Relative frequency

1. $54/100 = 0.54$.
2. number of times an event occurs $\div$ total trials.
3. $12/60 = 0.2$.
4. theoretical probability.
5. 0.4.
6. $6/200 = 0.03$.
7. 0.7.
8. 0.54 — close to 0.5, likely fair.
9. 0.75.
10. $28/80 = 0.35$.

14.3 Venn diagram (2 sets)

1. $A \cap B$ (both).
2. $20 + 15 - 5 = 30$.
3. 45.
4. $50 - 45 = 5$.
5. 11.
6. 14.
7. $40 - 32 = 8$.
8. $80/100 = 0.8$.
9. $(50-20)+(30-20) = 40$.
10. $21/30 = 0.7$.

14.4 Union and intersection

1. A or B (union).
2. A and B (intersection).
3. $\{1,2,3,4,5\}$.
4. $\{3\}$.
5. 15.
6. 60.
7. $40/100 = 0.4$.
8. mutually exclusive.
9. $15 + 14 - 25 = 4$.
10. $\{4, 6\}$.

14.5 Probability identities

1. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.
2. 0.7.
3. 0.9.
4. 0.1.
5. 0.
6. 0.55.
7. 0.4.
8. $(60 + 50 - 30)/100 = 0.8$.
9. $0.7 = 0.45 + P(B) - 0.15 \rightarrow P(B) = 0.4$.
10. Together they cover all outcomes (complementary).

14.6 Mutually exclusive events

1. happen at the same time.
2. Yes.
3. 0.
4. 0.8.
5. Yes (a card can't be both).
6. Yes.





7. $1 - 0.75 = 0.25$.
8. No (red face cards exist).
9. 0.85.
10. mutually exclusive.

14.7 Complementary events

1. 0.7.
2. 0.75.
3. $5/6$.
4. 1.
5. 0.45.
6. 70%.
7. $0.2 \times 200 = 40$.
8. 0.96.
9. $7/10$.
10. $49/50$.

End of Chapter 14 — Mixed Exercise

1. $\frac{1}{2}$.
2. number of times an event occurs $\div$ total trials.
3. 45.
4. $\{3\}$.
5. 0.
6. Yes.
7. $0.2 \times 200 = 40$.
8. $3/8$.
9. 0.75.
10. $21/30 = 0.7$.

CHAPTER 10 — Trigonometry

10.1 Defining the trigonometric ratios

1. opposite $\div$ hypotenuse.
2. adjacent $\div$ hypotenuse.
3. opposite $\div$ adjacent.
4. $3/5$.
5. $4/3$.
6. $8/10 = 4/5$.
7. adjacent = 4 $\rightarrow \cos \theta = 4/5$.
8. 0.6.
9. 45° .
10. $\sin 3/5$, $\cos 4/5$, $\tan 3/4$.

10.2 Reciprocal ratios

1. $\sin \theta$.
2. $\cos \theta$.
3. $\tan \theta$.
4. 2.
5. $5/4$.
6. $4/3$.
7. hypotenuse $\div$ opposite.
8. 2.
9. $1/2$.
10. $\tan \theta = 1 \rightarrow \theta = 45^\circ$.





10.3 Calculator skills

1. 0.5.
2. 0.5.
3. 1.
4. 1.
5. 1.
6. 0.8.
7. 0.58.
8. 30° .
9. 60° .
10. $\approx 60^\circ$ (60°).

10.4 Special angles

1. $1/2$.
2. $\sqrt{3}/2$.
3. 1.
4. $\sqrt{3}/2$.
5. $\sqrt{2}/2$ ($= 1/\sqrt{2}$).
6. $\sqrt{3}$.
7. $\sqrt{2}/2$.
8. $1/\sqrt{3}$.
9. $\frac{1}{2} + \frac{1}{2} = 1$.
10. $\frac{1}{2} + \frac{1}{2} = 1$.

10.5 Solving trigonometric equations

1. $\theta = 30^\circ$.
2. $\theta = 60^\circ$.
3. $\theta = 45^\circ$.
4. $\theta = 90^\circ$.
5. $\theta = 0^\circ$.
6. $\sin \theta = \frac{1}{2} \rightarrow \theta = 30^\circ$.
7. $\theta = 60^\circ$.
8. $\theta \approx 53.13^\circ$.
9. $\tan \theta = 1 \rightarrow \theta = 45^\circ$.
10. $\theta \approx 30^\circ$ (30°).

10.6 Ratios on the Cartesian plane

1. First (Q1).
2. $\sin$.
3. $\tan$.
4. $\cos$.
5. $\sqrt{9 + 16} = 5$.
6. $4/5$.
7. $3/5$.
8. $4/3$.
9. Q2; $\cos \theta$ negative.
10. Negative.

End of Chapter 10 — Mixed Exercise

1. opposite $\div$ hypotenuse.
2. $\cos \theta$.
3. 1.
4. $\sqrt{3}/2$.
5. $\theta = 0^\circ$.
6. $4/5$.



7. adjacent = 4 $\rightarrow \cos \theta = 4/5$.
8. 2.
9. 60° .
10. $\frac{1}{2} + \frac{1}{2} = 1$.

CHAPTER 11 — Functions

10.F1 Straight Line function

1. $m = 2$, intercept (0, 3).
2. (3, 0).
3. $y = 3x + 1$.
4. (2, 2).
5. $-\frac{1}{2}$.
6. R52.
7. $t = 10$ hours.
8. $m = 3 \rightarrow y = 3x + 1$.
9. $2(1) + 3 = 5 \rightarrow \text{yes}$.
10. (0, 6).

10.F2 Quadratic function

1. (0, -4).
2. $x = \pm 3$.
3. (0, 0).
4. Up ($a > 0$).
5. $x = 2$.
6. $x(4 - x) = 0 \rightarrow x = 0$ or 4.
7. (2, -1).
8. Max of 2 at $x = 0$.
9. $x = 2$ or 3.
10. $y = x^2 + 3$.

10.F3 Hyperbolic function

1. $x = 0$ and $y = 0$.
2. 3.
3. $x = 4$.
4. $x = 0, y = 2$.
5. 0.
6. 30 hours.
7. 24.
8. $x^2 = 4 \rightarrow (2, 2)$ and $(-2, -2)$.
9. $a = 12 \rightarrow y = 12/x$.
10. $x = 1, y = 0$.

10.F4 Exponential function

1. $y = 0$.
2. 8.
3. (0, 1).
4. Decreasing.
5. $y = -4$.
6. 32.
7. R1,259.71.
8. R144,500.00.
9. (0, 3).
10. 6.25 mg.



**End of Chapter 11 — Mixed Exercise**

1. $m = 2$, intercept $(0, 3)$.
2. $x = \pm 3$.
3. $x = 4$.
4. Decreasing.
5. $-\frac{1}{2}$.
6. $x(4 - x) = 0 \rightarrow x = 0$ or 4 .
7. 24.
8. R144,500.00.
9. $2(1) + 3 = 5 \rightarrow$ yes.
10. $y = x^2 + 3$.

CHAPTER 12 — Exponents**10.E1 Revision of exponent laws**

1. x^7 .
2. x^5 .
3. x^6 .
4. 1.
5. $8x^3$.
6. $2^7 = 128$.
7. x^3 .
8. $9x^4y^2$.
9. $1/x^3$.
10. a^5b^4 .

10.E2 Rational exponents

1. $x^{(1/2)}$.
2. 3.
3. 2.
4. $(16^{(1/4)})^3 = 2^3 = 8$.
5. 5.
6. 3.
7. $x^{(2/3)}$.
8. $(\sqrt{4})^3 = 8$.
9. $(\sqrt[5]{32})^2 = 2^2 = 4$.
10. x .

10.E3 Exponential equations

1. $x = 3$.
2. $x = 4$.
3. $x = 5$.
4. $x = 0$.
5. $x = 4$.
6. $x = 3$.
7. $x + 1 = 4 \rightarrow x = 3$.
8. $2x = 4 \rightarrow x = 2$.
9. $t = 6$.
10. $4^x = 4^3 \rightarrow x = 3$.



**End of Chapter 12 — Mixed Exercise**

1. x^7 .
2. 3.
3. $x = 5$.
4. 1.
5. 5.
6. $x = 3$.
7. x^3 .
8. $(\sqrt{4})^3 = 8$.
9. $t = 6$.
10. a^5b^4 .

CHAPTER 13 — Number Patterns**10.P1 Arithmetic sequences**

1. 19.
2. $d = 4$.
3. 29.
4. $T_n = 5n - 4$.
5. 61.
6. 56.
7. R10500.
8. $4 + (n-1)3 = 100 \rightarrow n = 33$.
9. $T_n = 10 - 3n$.
10. $d = 4, a = 3$.

End of Chapter 13 — Mixed Exercise

1. 19.
2. $d = 4$.
3. 29.
4. $T_n = 5n - 4$.
5. 61.
6. 56.
7. R10500.
8. $4 + (n-1)3 = 100 \rightarrow n = 33$.
9. $T_n = 10 - 3n$.
10. $d = 4, a = 3$.





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